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Sequential Non-overlapping Random Packing of Disks in a Square Box

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ABSTRACT

The total area occupied by equal disks randomly placed in a square region, one after another and without overlapping is investigated by means of Monte Carlo simulation, having as a reference the orderly placement of disks in the same region. It is found that the occupied area ranges from 56% to 65% of the area occupied if the disks were orderly placed.

Keywords: Random, disk packing, Monte Carlo, simulation.

EMPAQUE SECUENCIAL ALEATORIO NO SUPERPUESTO DE DISCOS EN UNA CAJA CUADRADA

RESUMEN

Mediante una simulación Monte Carlo, se investiga el área total ocupada por discos iguales, colocados al azar uno después del otro y sin traslaparse, en una región cuadrada, teniendo como referencia el empaque ordenado de discos en la misma región. Se encuentra que el área ocupada está entre el 56% y 65% del área que sería ocupada si la región se llenara ordenadamente.

Palabras clave: Aleatorio, empaque de discos, Monte Carlo, simulación.

INTRODUCTION

Sometimes it is necessary to estimate the number of elements occupying a given rectangular area; one such situation is when in a factory there are cylindrical cans standing on the floor, touching each other, another situation is when there are people attending a concert in an open arena, where individuals are just jammed one beside another. In cases like these, when it is impossible to count, it is necessary to resort to simulation and the investigation being reported here tries to find an answer to this dilemma.

The problem of random packing of disks on a plane surface[1-5] and of spheres inside a volume has been studied by many researchers, each one dealing with the problem in a particular manner.

Counting the number of disks orderly packed in a rectangular box is an easy task as it can be seen in Fig. (1) which shows two instances of disks packed in organized ways; The difficult task arises when trying to figure out the number of disks inside a rectangular region when they are randomly packed one beside another and without overlapping, in this situation the counting must be necessarily by means of simulation and the result is merely statistical.

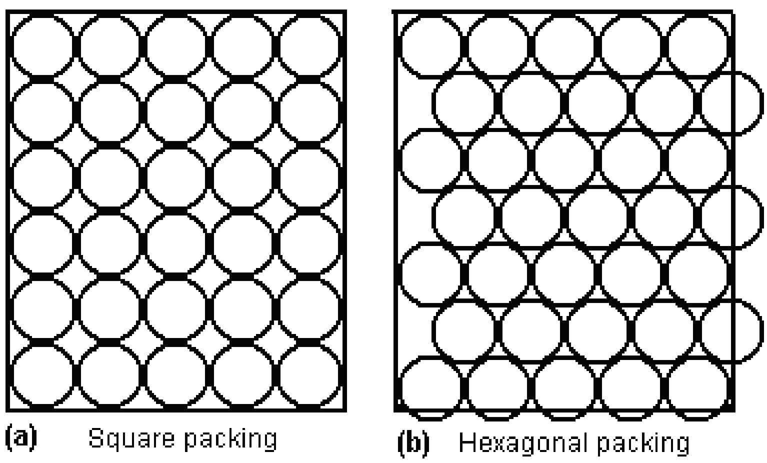

Fig. 1

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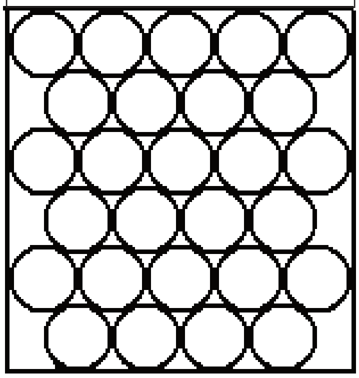
It can be seen in Fig. 1 that square packing (Fig 1-a) is more efficient than hexagonal packing (Fig 1-b), this is, for a rectangular region whose sides are integer multiples of the circle diameter, there are more circles packed in the square packing mode (Fig 2).

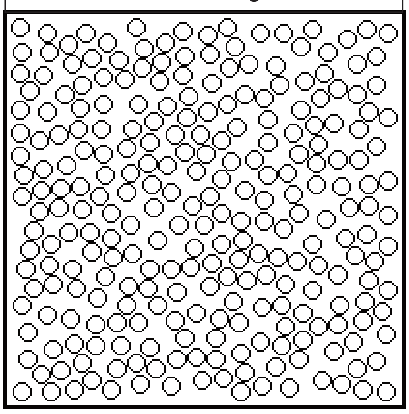
Fig. 2. Hexagonal packing in a tight rectangular box



In this research square regions whose side lengths are integer multiples of diameters of circles hexagonally packed (like in Fig 2) are considered and the maximum number of circles of a given diameter that can be randomly packed without overlapping is calculated. The corresponding fully occupied area like in Fig 2 is normalized to 1 (or 100%), therefore when randomly packing the result must be a fraction of 1. Disk packing at random will very hardly achieve the configuration shown in Fig. 2, whose density is 1.

At simulation time circles of the same area are randomly placed one after another in any position in the region, without overlapping, like shown in Fig. 3.

Fig. 3. Sequential random packing of circles of diameter 10 in a 200 x 200 region



PARAMETERS OF THE INVESTIGATION

- The packing simulation was made on square regions whose side lengths are integer multiples of diameters of circles hexagonally packed.

- For every simulation a fixed area region and a fixed circle diameter were used, however, region and circle areas were not the same throughout the investigation.

- One hundred packing simulations were made for every set of parameters (fixed region area and fixed circle diameter).

- Since the small zones between packed circles can not be filled, the area occupied by circles was considered that of a square having sides equal to the diameter of the circle, this is the area of every circle of diameter d was considered as d2.

- Since finding a site where to place a circle becomes more and more difficult as the number of packed circles increases, the number of trials for every simulation was 250 times that of the number of circles needed to orderly pack them in a given region. For instance, in the case of a region with area AR = 500x500 and circles with area Ac = 100, the maximum possible number of orderly packed circles is AR / Ac = 250000 / 100 = 2500. In this case the number of trials was 250x2500 = 625000, this is, in this case, every one of the 100 simulations stopped (whatever the number of already packed circles) after 625000 attempts.

- The relation R is established to compare the packings, it is given by:
  R = N packed Circles / Max Possible Packings
  and in the case of orderly packing, R = 1.00 It is expected that in the case of random packing R becomes a fraction. As an example, consider the case of an area of 500x500 and circles of area 100: In this case R = 1 when the number of packed circles is 2500.

RESULTS OF THE INVESTIGATION

For a given packing region, the larger the diameter of the circles, the smaller the number of circles packed.

For every square and circle area investigated, a number of 100 simulations were executed and an average R was found. As an example Fig. 4 displays the results obtained from 100 simulations for a region area of 500x500 where circles of area 16 were randomly packed, in this case the average filling was 61%.

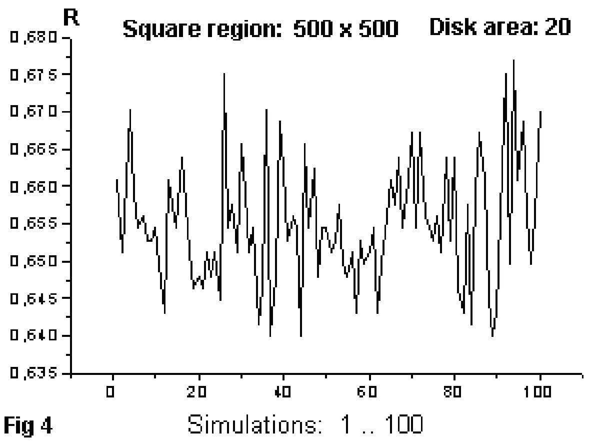


Fig 4

A number of 16 different combinations of packing and circle areas were investigated, in each case 100 Monte Carlo packing simulations were performed and each one of the 16 instance reported an average R, these 16 average values were in general different, having a minimum and a maximum.

The minimum average R found was 0.5687 and the maximum was 0.6554 (Table 1), it can be concluded that the area filled ranges from 56% to 65% of the area that would be filled if the packing were made in an orderly mode.

The fact that the length and width of the region where circles are placed must make exact room for an integer number of them, imposed a limitation to this research since it reduced the cases that were investigated.

CONCLUSIONS

The larger the circles, the smaller the value of R, this is due to the fact that the larger the circle, the larger the area that remains empty after parking.

The area filled ranges from 56% to 65% of the area that would be filled if the packing were in an orderly regime, this might lead to the conclusion that when disks are randomly placed in a square box without overlapping, the area filled by the disks must be approximately 60% of the region area; this approximation may result useful when trying to determine the number of people in a crowd, like when trying to estimate the number of persons attending a concert in an open ground.

In this research the maximum number of trials was 250 times that of the maximum number of orderly placed circles. It is expected that more accurate results -but not too different- will be obtained with a higher number of trials. Also it is expected that the same result will be obtained when dealing with rectangular packing areas.

Table 1
Sequential Non-overlapping Random Packing of Disks in a Square Box
Average values of R.
R = Packed Circles / Max Possible Packings

	Diameter: 4	**Diameter: 10**	**Diameter: 20**	**Diameter: 30**
Region Size				
100 x 100	0.6395	0.6237	0.5988	
200 x 200		0.6431	0.6293	
300 x 300		0.6485	0.6452	
400 x 400		0.6508	0.6515	
500 x 500		0.6532	0.6554	
120 x 120				0.5687
210 x 210				0.6096
300 x 300				0.6383
390 x 390				0.642
510 x 510				0.6482

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