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Alternative demonstration of the entropy evolution in least time

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Open systems evolve towards states of greater entropy, so they come into balance with their surroundings. Also, this evolution occurs in least time. This work presents a mathematical demonstration of this principle.
Keywords: open system, entropy, least time..

Demostración alternativa de la evolución de la entropía en el menor tiempo

Los sistemas abiertos evolucionan hacia estados de mayor entropía, por lo que entran en equilibrio con sus alrededores. Asimismo, esta evolución se da en el menor tiempo. Este trabajo presenta una demostración matemática de este principio.
Palabras claves: sistema abierto, entropía, menor tiempo..

1. Introduction

In recent years, a new definition of the entropy concept has been proposed. According to this new vision, an open system with high entropy would have low potential energy and equilibrium with its surroundings; in the same way, a system with low entropy would have high potential energy and it will be evolve to a high entropy state, so that it would be in equilibrium with its surroundings. In this way, this new definition allows the development of postulates that explain the evolution of open systems [1], [2], [3]. In this work a mathematical demonstration of the minimum time principle in the evolution of entropy is presented.

2. Entropy evolution in least time

The variation of the entropy of an open system, in a spontaneous and irreversible process, can be described as [4]:

$$\Delta S_{univ} = \Delta S_{sist} + \Delta S_{alr} > 0 \tag{1}$$

Where ΔS_{univ} is the variation of entropy in the universe, ΔS_{sist} is the variation of the entropy of an open system and ΔS_{alr} is the variation of the entropy of the surroundings [4].

Therefore, the variation of the entropy at a time t can be described as [4]:

$$\frac{dy}{dx} > 0 \tag{2}$$

If we represent the entropy (S) as a function of time, we would have entropy on the vertical axis and time (t) on the horizontal axis. Therefore, if we have two points (t1; S1) and (t2; S2), the distance between these two points would be (Figure 1) [5]:

$$D = \int_{t_1}^{t_2} \sqrt{1 + F'^2}\, dt \tag{3}$$

Where t is the time and $F' = dy/dx$ [5].

If we solve equation (3), we have:

$$\begin{aligned} dD &= \sqrt{1+F'^2}dt \\ (dD)^2 &= (1+F'^2)(dt)^2 \\ -F'^2(dt)^2 &= (dt)^2 - (dD)^2 \\ -F'^2 &= \frac{(dt)^2-(dD)^2}{(dt)^2} \end{aligned}$$

$$F' = \sqrt{-1 + \frac{(dD)^2}{(dt)^2}} \tag{4}$$

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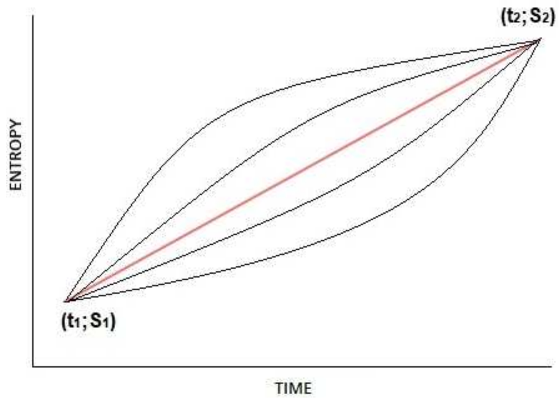

Figure 1: We can see the different curves that join the points ($t1$; $S1$) and ($t2$; $S2$). However, these points only can be joined with a straight line (pink line).

Replacing (4) in (2), we have:

$$F' = \sqrt{-1 + \frac{(dD)^2}{(dt)^2}} > 0 \tag{5}$$

If we know that F' belonging to $\mathbb{R}$ [5]; therefore:

$$\frac{(dD)^2}{(dt)^2} > 1 \tag{6}$$

$$\frac{(dD)}{(dt)} > 1 \vee \frac{(dD)}{(dt)} < -1$$

Therefore, we have:

$$dD = dt.k \tag{7}$$

Where k is a constant and depends on the entropy value. Replacing (7) in (3), we have:

$$D = \int_{t_1}^{t_2} \sqrt{1 + F'^2}\, dt = \int_{t_1}^{t_2} k\, dt$$

Then:

$$k = \sqrt{1 + F'^2}$$
$$\sqrt{k^2 - 1} = F'$$
$$k^2 - 1 = F'^2 k^2 - F'^2 (k^2 - 1)$$
$$(1 + F'^2)(k^2 - 1) = (F'^2)(k^2)$$
$$\frac{k^2 - 1}{k^2} = \frac{F'^2}{1 + F'^2}$$
$$1 - \frac{1}{k^2} = \frac{F'^2}{1 + F'^2}$$
$$\sqrt{1 - \frac{1}{k^2}} = \frac{F'}{\sqrt{1 + F'^2}}$$

$$\frac{F'}{\sqrt{1 + F'^2}} = \alpha \tag{8}$$

Where α is a constant, rearranging the equation (8):

$$F' = \frac{\alpha}{\sqrt{1 - \alpha^2}} \tag{9}$$

Therefore, we have:

$$\frac{d}{dt}\left(\frac{F'}{\sqrt{1 + F'^2}}\right) = 0 \tag{10}$$

Equation (10) is satisfied only if the curve joining the points (t1; S1) and (t2; S2) is a straight line [5], therefore the time an open system takes in a spontaneous and irreversible process to evolve towards one of highest entropy is always the minimum.

Therefore, the equation of the line would be [5]:

$$S = mt + b \tag{11}$$

Where m is the slope of the straight and b is a constant [5]. We would have that slope m is constant and greater than zero.

3. Conclusion

The evolution of entropy, from open systems with low entropy to systems with high entropy, always occurs in the shortest possible time.

4. Acknowledgment

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