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# The Terminal Velocity of Compound Drops in Flotation Units: Effect of Inner Bubble Size

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**Resumen**

El comportamiento de la velocidad terminal de las gotas compuestas gas-líquido se estudió experimentalmente. Se analizó el efecto del tamaño de la burbuja interna, cubriendo un amplio rango de tamaños relativos. La burbuja aumentó significativamente los efectos de flotabilidad, particularmente para fluidos externos más densos. La velocidad terminal aumenta con el tamaño de la burbuja interna, lo que sugiere que el movimiento está gobernado por la geometría de la gota compuesta. Se propuso un modelo mecanicista que mostró una buena concordancia con las mediciones.
**Palabras clave**: Flotación, Gota compuesta, Velocidad terminal, Eficiencia.

**Abstract**

The behavior of the terminal velocity of gas-liquid compound drops was studied experimentally. The effect of the size of the internal bubble was analyzed, covering a wide range of relative sizes. The bubble significantly increased the buoyancy effects, particularly for denser external fluids. The terminal velocity increases with the size of the internal bubble, suggesting that the motion is governed by the geometry of the compound drop. A mechanistic model was proposed, and it showed a good agreement with the measurements.
**Keywords**: Flotation, Compound drop, Terminal velocity, Efficiency.

## Introduction

The terminal velocity of a bubble or drop is a crucial parameter in the design of gravitational settling tanks [Kha12]. Flotation units serve as an example of these separators [Sat16]. The technique aims separating oil drops dispersed in produced water in oil processing sites by the insertion of gas bubbles [Sar05]. The contribution of the gas bubbles can either be given by:

- Formation of gas-liquid compound drops;
- Drafting of the drop by the bubble;

This work focuses on the first hypothesis, according to the schematics shown in Fig. 1. The bubble collides with the drop, increasing its terminal velocity by forming a bubble-drop compound [Pai05]. Clearly, this process depends on the size of the particles and the properties of the fluids [Oli99].

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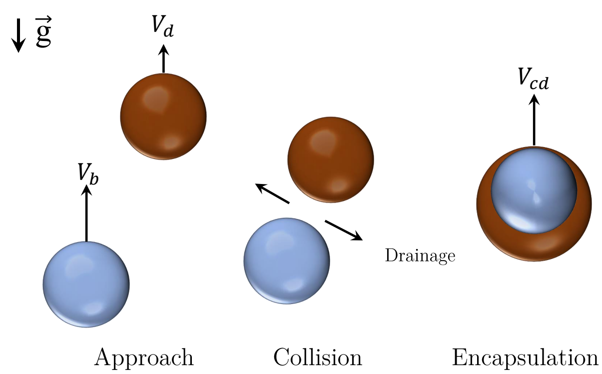


**Figure 1**: Formation of gas-liquid compound drops in flotation units.

The theory of translating compound drops has been presented by [Rus83], [Joh83], [Bru85]. An intermediate behavior between the Stokes and Hadamard & Rybczynscki solutions [Cli78] is proposed when inertia is absent. Experiments by [Hay74] suggest that the volume ratio is the governing parameter of the motion of compound drops. Recently, [Wan18] showed that an oil-coated bubble presents intermediate behavior compared to single-fluid drops and bubbles of correspondent size. However, the effect of the relative size was not addressed.

In this work, we analyse the terminal velocity of compound drops as a function of the size of the internal bubble. A model to predict the velocity of compound drops when inertial forces are relevant is herein proposed, and a good agreement.

## 1. Description of experiments

### 1.1. Experimental setup

The experiments were conducted in a glass tank ($500 \times 150 \times 150$ mm) filled with tap water. The drops and bubbles were generated by the pinch-off method from two capillary tubes placed at the bottom of the chamber. The diameter of the internal bubble varied from 0.3 to 3.0 mm, whereas the diameter of the drop was 2.5 mm. The compound drop was formed by approaching the capillaries. A high-speed camera (Photron SA4) was used to study the motion of the compound drops. Illumination was supplied by a LED panel coupled to the experimental setup.

Nitrogen (99.8 % purity) was used for generating the bubble. Three different liquids made the drops, namely, Oil 1 (silicone oil, 47V350 Rhodorsil), Oil 2 (commercial vegetable oil), and Oil 3 (mineral oil, 330779 Sigma-Aldrich). The physical properties of these fluids are summarized in Table 1.

| Fluid | $\rho$ | $\mu$ | $\sigma_{o/w}$ | $\sigma_{o/g}$ | $k$ | $n$ |
|---|---|---|---|---|---|---|
| Tap water | 998 | 1.002 | - | 73.5 | - | - |
| Oil 1 | 970 | 350 | 41.1 | 21.1 | 7.8 | 1.3 |
| Oil 2 | 915 | 59 | 26.0 | 31.6 | 2.7 | 0.6 |
| Oil 3 | 838 | 13 | 18.0 | 28.6 | 2.4 | 0.55 |

**Table 1**: Properties of the fluids used in the experiments. $\rho$ is the density in $kg/m^3$, $\mu$ is the viscosity in $Pa \cdot s$, and $\sigma_{o/w}$ and $\sigma_{o/g}$ are the oil-water and oil-gas interfacial tensions in $mN/m$, respectively. The values were taken from [Ga078], and [Hua97]. $k$ and $n$ experimental parameters used in Eq. 4.

### 1.2. Image processing

The images were processed using MATLAB® according to a scheme shown in Fig. 2. Both the internal and external contours were detected by image binarization.

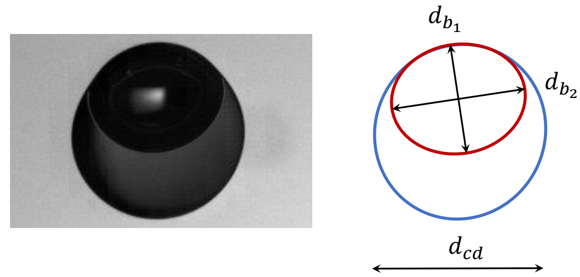


**Figure 2**: (left) Raw image (right) Processed image.

The equivalent diameter of the compound drop, $d_{cd}$, is calculated by considering a spherical shape. The internal bubble is ellipsoidal and its equivalent diameter, $d_b$, is calculated as [Leg12]

$$d_b = \left(d_{b1} \cdot d_{b2}^2\right)^{1/3} \quad (1)$$

where $d_{b1}$ and $d_{b2}$ are the short and large axes, respectively. A working density $\rho_{cd}$ is defined based on the volume fraction of each fluid in the compound drop

$$\rho_{cd} = \rho_o \left(1 - d_*^3\right) \quad (2)$$

where $\rho_o$ is the density of the oil, and $d_* = d_b/d_{cd}$ is the normalized diameter.

## 2. Results and Discussion

### 2.1. Compound drop properties

We begin by analyzing the motion parameters when the size of the bubble changes. Figure 3 shows the Morton number $Mo = g\left(\rho - \rho_{cd}\right)/\rho^2\sigma_{o/w}^3$ of the compound drops as a function of the normalized diameter $d^*$. Note that the bubble is considered in the term $\rho_{cd}$. The influence of the bubble is small when $d^* < 0.2$ and the measurements of $Mo$ are equal to those of the original drop of the same fluid. However, once the bubble reaches a certain size ($d^* > 0.4$) the Morton number begins to escalate with $d^*$. Thus, buoyancy effects increase, which may lead to a higher terminal velocity.

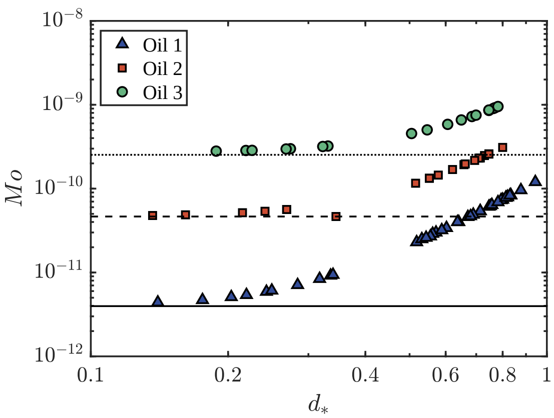


**Figure 3**: Evolution of the Morton number $Mo$ of the compound drops as a function of the normalized diameter $d^*$. Solid, dashed, and dotted lines represent the Morton number of the original drops for Oils 1, 2, and 3, respectively.

The next step is to quantify the inertial effects. Figure 4 shows the Reynolds number $Re = \rho V_{cd} d_{cd}/\mu$ against the Weber number $We = \rho V_{cd}^2 d_{cd}/\sigma_{o/w}$. The numbers are analyzed in terms of the external fluid. $Re$ is modified by taking the viscosity ratio $\mu^* = (\mu_o/\mu)^{1/5}$ into account. Similarly, $We$ is multiplied by the density ratio $\rho^* = \rho_o/(\rho - \rho_o)$. From Fig. 4, all measurements can be fitted by the correlation

$$Re \times \mu^* = 150\,(We \times \rho^*)^{0.55} \tag{3}$$

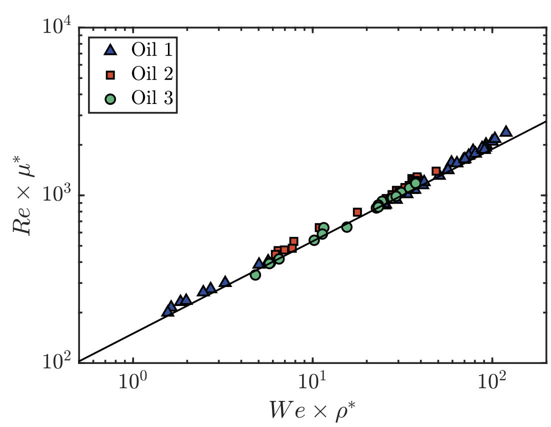


**Figure 4**: Evolution of the Reynolds number $Re$ with the Weber number $We$ for all cases. The solid line represents Eq. 3.

## 2.2. Terminal velocity of a compound drop

To measure the relevance of the bubble in the motion we define a normalized velocity $V^* = V_{cd}/V_d$, plotted against $d^*$ in Fig. 5. As expected, $V^*$ rapidly increases when $d^* > 0.2$. For Oil 1, $V^*$ reaches a maximum of approximately 7 for higher, $d^*$ suggesting that the increase in terminal velocity is more pronounced for denser external fluids. Based on the power law described by Eq. 3, an expression for $V^*$ can be provided

$$V^* = k\,(d^*)^n \tag{4}$$

where $k$ and $n$ are given in Table 1.

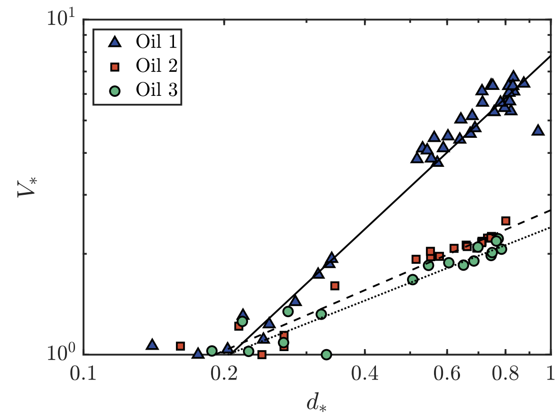


**Figure 5**: Normalized terminal velocity of the compound drops $V^*$ as a function of the normalized diameter $d^*$. Solid, dashed, and dotted lines represent Eq. 4 for Oil 1, 2, and 3, respectively.

We now propose a mechanistic model to predict $V^*$ for spherical compound drops. If viscous forces are negligible

$$V^* = \left[\frac{d_d^3}{d_b d_{cd}^2}\,(A^* + 1)\right]^{1/2} \tag{5}$$

where $d_d = \left(d_{cd}^3 - d_b^3\right)^{1/3}$ is the diameter of the original drop and $A^*$ is the normalized Archimedes force defined as $A^* = (A_{cd} - A_d)/A_d$. The subscripts $cd$ and $d$ refer to the compound and to the original drop, respectively. The Archimedes force is calculated by a balance with the drag force

$$A_{cd} = \frac{1}{6}\pi d_{cd}^3 g\,(\rho - \rho_{cd}) \tag{6}$$

and

$$A_d = \frac{1}{6}\pi\left(d_{cd}^3 - d_b^3\right) g\,(\rho - \rho_o) \tag{7}$$

A model for $A^*$ is provided by combining Eqs. 2, 6, and 7

$$A_d = \rho^* \left[\frac{d_b}{d_d}\right]^3 \tag{8}$$

Figure 6 confirms that $V^*$ is proportional to the increase in the Archimedes force caused by the bubble.

Moreover, this effect is more pronounced when the density of the external fluids is large. Finally, Fig. 6 shows a good agreement of Eq. 5 with the experiments. Thus, this simple model is able to predict the increase in terminal velocity promoted by the internal bubble.

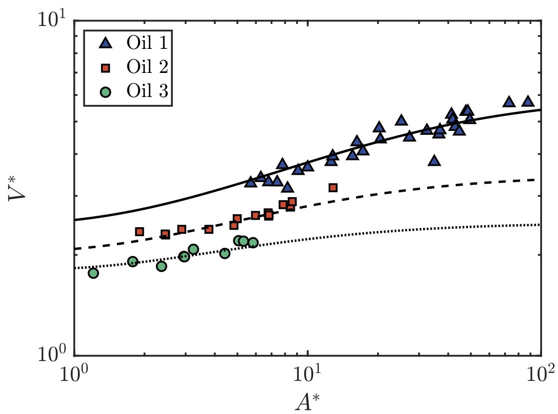


**Figure 6**: Normalized velocity $V^*$ as a function of the normalized Archimedes force $A^*$. Solid, dashed, and dotted lines represent Eq. 5 for Oils 1, 2, and 3, respectively.

## 3. Conclusions

In the present study, the terminal velocity of compound drops has been analyzed experimentally. We focus on the effect of the size of the internal bubble. The Morton number increases monotonically with the normalized diameter $d^*$, suggesting that the compound drops experience a more pronounced Archimedes force. Thus, the terminal velocity is increased. A mechanistic model based on a normalized Archimedes force has been proposed and it led to a good agreement.

The experiments conducted here have straight applicability in enhanced primary oil separation processes. We conclude that the addition of a gas bubble inside an oil drop increases the efficiency of separation, particularly for critical situations where the density of the drop and the one of the continuous phase are close.

## 4. Acknowledgements

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