CORRIGENDUM

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Corrigendum: A relativistic theory of the field

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Introduction

In writing out the eq. (36) for $\ln\sqrt{-g}$ in my recent paper [1], an inadvertent error led to two errant numerical expressions in equations (37) and (38) that propagated to the "Field equations". The main results and conclusions of my paper remain unchanged. All of the affected formulas are corrected below.

Field equations

We first construct a new tensor by substracting a certain tensor $S_{\mu\nu}$ from

$$R'_{\mu\nu} = \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^{\rho}} - \frac{1}{2}\left(\frac{\partial \Gamma^{\rho}_{\mu\rho}}{\partial x^{\nu}} + \frac{\partial \Gamma^{\rho}_{\rho\nu}}{\partial x^{\mu}}\right) - \Gamma^{\lambda}_{\mu\rho}\Gamma^{\rho}_{\lambda\nu} + \\ + \frac{1}{2}\Gamma^{\lambda}_{\mu\nu}\left(\Gamma^{\rho}_{\lambda\rho} + \Gamma^{\rho}_{\rho\lambda}\right) \tag{1}$$

According to

$$\frac{1}{2}g^{\mu\nu}\nabla_{\eta} g_{\underset{+}{\mu}\underset{-}{\nu}} = \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial x^{\eta}} - \frac{1}{2}\left(\Gamma^{\lambda}_{\eta\lambda} + \Gamma^{\lambda}_{\lambda\eta}\right); \tag{2}$$

we have that

$$S_{\eta} = \frac{\partial\left(\log\sqrt{-g}\right)}{\partial x^{\eta}} - \frac{1}{2}\left(\Gamma^{\lambda}_{\eta\lambda} + \Gamma^{\lambda}_{\lambda\eta}\right) \tag{3}$$

is a vector. From it, we construct the tensor $\nabla_{\nu} S_{\underset{+}{\mu}}\ (= S_{\mu\nu})$ getting

$$S_{\mu\nu} = \frac{\partial^2\left(\log\sqrt{-g}\right)}{\partial x^{\nu}\partial x^{\mu}} - \frac{\partial\left(\log\sqrt{-g}\right)}{\partial x^{\alpha}}\Gamma^{\alpha}_{\mu\nu} - \frac{1}{2}\times \\ \times\left[\frac{\partial}{\partial x^{\nu}}\left(\Gamma^{\lambda}_{\lambda\mu} + \Gamma^{\lambda}_{\mu\lambda}\right) - \left(\Gamma^{\lambda}_{\lambda\alpha} + \Gamma^{\lambda}_{\alpha\lambda}\right)\Gamma^{\alpha}_{\mu\nu}\right] \tag{4}$$

We get

$$R^{*}_{\mu\nu} = R'_{\mu\nu} - S_{\mu\nu} = \frac{\partial}{\partial x^{\sigma}}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \\ - \frac{\partial^2\left(\log\sqrt{-g}\right)}{\partial x^{\mu}\partial x^{\nu}} + \frac{\partial\left(\log\sqrt{-g}\right)}{\partial x^{\lambda}}\Gamma^{\lambda}_{\mu\nu} \tag{5}$$

From this we construct with the help of the tensor-density $\sqrt{-g}g^{\mu\nu}$ the Lagrangian density-function

$$\mathscr{L} = \sqrt{-g}g^{\mu\nu}R^{*}_{\mu\nu}. \tag{6}$$

The variation of the integral of $\mathscr{L}$ with respect to $\Gamma^{\lambda}_{\mu\nu}$ and $\sqrt{-g}g^{\mu\nu}$ yields

$$\delta\int\mathscr{L}d\tau = \int d\tau\,\delta\left(\sqrt{-g}g^{\mu\nu}\right)R^{*}_{\mu\nu} - \int d\tau\left[\frac{\partial^2}{\partial x^{\nu}\partial x^{\mu}}\left(\sqrt{-g}g^{\mu\nu}\right) + \frac{\partial}{\partial x^{\lambda}}\left(\sqrt{-g}g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}\right)\right]\delta\left(\log\sqrt{-g}\right) \\ - \int d\tau\left[\frac{\partial}{\partial x^{\sigma}}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\lambda\nu}\Gamma^{\mu}_{\lambda\sigma} + \sqrt{-g}g^{\mu\lambda}\Gamma^{\nu}_{\sigma\lambda} - \sqrt{-g}g^{\mu\nu}\frac{1}{2g}\frac{\partial g}{\partial x^{\sigma}}\right]\delta\Gamma^{\sigma}_{\mu\nu} \tag{7}$$

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The variation with respect to $\sqrt{-g}g^{\mu\nu}$ yields

$$G_{\mu\nu}\delta\left(\sqrt{-g}g^{\mu\nu}\right)=\delta\left(\sqrt{-g}g^{\mu\nu}\right)R^{*}_{\mu\nu}-\left[\frac{\partial^{2}\left(\sqrt{-g}g^{\mu\nu}\right)}{\partial x^{\nu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\lambda}}\left(\sqrt{-g}g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}\right)\right]\delta\left(\log\sqrt{-g}\right)$$

where

$$\delta\left(\log\sqrt{-g}\right)=\frac{1}{2\sqrt{-g}}g_{\mu\nu}\delta\left(\sqrt{-g}g^{\mu\nu}\right).$$

Substituting this expression, we get as the result of variation with respect to $\sqrt{-g}g^{\mu\nu}$

$$G_{\mu\nu}=R^{*}_{\mu\nu}-\frac{1}{2\sqrt{-g}}\times \left[\frac{\partial^{2}\left(\sqrt{-g}g^{\rho\sigma}\right)}{\partial x^{\rho}\partial x^{\sigma}}+\frac{\partial}{\partial x^{\lambda}}\left(\sqrt{-g}g^{\rho\sigma}\Gamma^{\lambda}_{\rho\sigma}\right)\right]g_{\mu\nu} \tag{8}$$

The field equations following from our variational principle are then

$$\nabla_{\eta}\left(\sqrt{-g}g^{\overset{\mu\nu}{+-}}\right)=0 \tag{9}$$

$$G_{\mu\nu}=0. \tag{10}$$

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References

[1] M. Valenzuela. A relativistic theory of the field, Revista de Investigación de Física, **24**(**2**), 72-79 (2021). Doi: https://doi.org/10.15381/rif.v24i2.14245