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Corrigendum: A relativistic theory of the field II

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Introduction

In writing out the equations in section of Hamilton principle in my recent paper [1], inadvertents errors led to errant numerical expressions in "Field equations". The main results and conclusions of my paper remain unchanged. All of the affected formulas are corrected below.

Hamiltonian principle. Field equations

In the case of the real symmetric field we obtain the fields equations most simply in the following manner. We use as Lagrangian function the scalar density

$$\mathscr{L}_G = \sqrt{-g} g^{\mu\nu} R_{\mu\nu} \tag{1}$$

where

$$R_{\mu\nu} = \frac{\partial}{\partial x^\rho}\Gamma^\rho_{\mu\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\lambda\rho} - \frac{\partial}{\partial x^\nu}\Gamma^\rho_{\mu\rho} - \Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\lambda\nu}$$

is the curvature tensor in the relativistic theory of gravitation.

If we vary the volume integral of $\mathscr{L}_G$, i. e.

$$\begin{aligned}\delta \int \mathscr{L}_G d\tau = &-\int d\tau \left[\tfrac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\lambda\nu}\Gamma^\mu_{\lambda\rho} + \sqrt{-g}g^{\mu\lambda}\Gamma^\nu_{\rho\lambda} - \sqrt{-g}g^{\mu\nu}\Gamma^\lambda_{\rho\lambda}\right]\delta\Gamma^\rho_{\mu\nu} \\ &+ \int d\tau \delta^\nu_\rho \left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\lambda\sigma}\Gamma^\lambda_{\lambda\sigma}\right]\delta\Gamma^\rho_{\mu\nu} - \int d\tau\sqrt{-g}\left(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R\right)\delta g^{\mu\nu}\end{aligned}$$

independiently with respect to Γ and g, then variation with respect to Γ yields

$$\begin{aligned}&-\left[\tfrac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\lambda\nu}\Gamma^\mu_{\lambda\rho} + \sqrt{-g}g^{\mu\lambda}\Gamma^\nu_{\rho\lambda} - \sqrt{-g}g^{\mu\nu}\Gamma^\lambda_{\rho\lambda}\right] \\ &+\delta^\nu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\lambda\sigma}\Gamma^\lambda_{\lambda\sigma}\right] = 0\end{aligned}$$

or $\frac{\partial g_{\mu\nu}}{\partial x^\rho} - g_{\alpha\nu}\Gamma^\alpha_{\mu\rho} - g_{\mu\alpha}\Gamma^\alpha_{\rho\nu} = 0$, and variation with respect to g yields the equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0$, or $R_{\mu\nu} = 0$. If we apply the same method in the relativistic theory of the field

$$\begin{aligned}\delta\int\mathscr{L}d\tau = &-\int d\tau\left[\tfrac{\partial}{\partial x^\rho}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\alpha\nu}\Gamma^\mu_{\alpha\rho} + \sqrt{-g}g^{\mu\alpha}\Gamma^\nu_{\rho\alpha} - \tfrac{1}{2}\sqrt{-g}g^{\mu\nu}\left(\Gamma^\lambda_{\rho\lambda} + \Gamma^\lambda_{\lambda\rho}\right)\right]\delta\Gamma^\rho_{\mu\nu} \\ &+\tfrac{1}{2}\int d\tau\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\mu\lambda}\right) + \sqrt{-g}g^{\alpha\lambda}\Gamma^\mu_{\alpha\lambda} - \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\right]\delta^\nu_\rho\delta\Gamma^\rho_{\mu\nu} \\ &+\tfrac{1}{2}\int d\tau\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\lambda\nu}\right) + \sqrt{-g}g^{\lambda\alpha}\Gamma^\nu_{\lambda\alpha} + \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\right]\delta^\mu_\rho\delta\Gamma^\rho_{\mu\nu} \\ &+\tfrac{1}{2}\int d\tau\left(\sqrt{-g}g^{\mu\alpha}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\nu_\rho - \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\mu_\rho\right)\delta\Gamma^\rho_{\mu\nu} + \int d\tau\delta\left(\sqrt{-g}g^{\mu\nu}\right)R_{\mu\nu}\end{aligned}$$

with $\mathscr{L} = \sqrt{-g}g^{\mu\nu}R^*_{\mu\nu}$ and

$$R^*_{\mu\nu} = \frac{\partial\Gamma^\rho_{\mu\nu}}{\partial x^\rho} - \frac{1}{2}\left(\frac{\partial\Gamma^\rho_{\mu\rho}}{\partial x^\nu} + \frac{\partial\Gamma^\rho_{\rho\nu}}{\partial x^\mu}\right) - \Gamma^\lambda_{\mu\rho}\Gamma^\rho_{\lambda\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\rho_{\underline{\lambda\rho}}. \tag{2}$$

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Then we see a complication, since the variation with respect to Γ does not inmediately yield the equation

$$\nabla_\rho g_{\underset{+-}{\mu\nu}} = \frac{\partial g_{\mu\nu}}{\partial x^\rho} - g_{\alpha\nu}\Gamma^\alpha_{\mu\rho} - g_{\mu\alpha}\Gamma^\alpha_{\rho\nu} = 0. \tag{3}$$

which we wish to keep in any case. The variation with respect to Γ yields

$$\begin{aligned} -\left[\tfrac{\partial}{\partial x^\rho}\left(\sqrt{-g}g^{\mu\nu}\right) + \sqrt{-g}g^{\mu\alpha}\Gamma^\nu_{\rho\alpha} + \sqrt{-g}g^{\alpha\nu}\Gamma^\mu_{\alpha\rho} - \tfrac{1}{2}\sqrt{-g}g^{\mu\nu}\left(\Gamma^\lambda_{\rho\lambda} + \Gamma^\lambda_{\lambda\mu}\right)\right] \\ +\tfrac{1}{2}\delta^\nu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\mu\lambda}\right) + \sqrt{-g}g^{\alpha\lambda}\Gamma^\mu_{\alpha\lambda} - \sqrt{-g}g^{\mu\alpha}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\right] \\ +\tfrac{1}{2}\delta^\mu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\lambda\nu}\right) + \sqrt{-g}g^{\lambda\alpha}\Gamma^\nu_{\lambda\alpha} + \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\right] \\ +\tfrac{1}{2}\left(\sqrt{-g}g^{\mu\alpha}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\nu_\rho - \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\mu_\rho\right) = 0. \end{aligned} \tag{4}$$

The first bracket is $\nabla_\rho\left(\sqrt{-g}g^{\overset{\mu\nu}{+-}}\right)$; the second and third brackets are contractions of this quantity, i. e.

$$\begin{aligned} -\nabla_\rho\left(\sqrt{-g}g^{\overset{\mu\nu}{+-}}\right) + \tfrac{1}{2}\delta^\nu_\rho\nabla_\lambda\left(\sqrt{-g}g^{\overset{\mu\lambda}{+-}}\right) + \tfrac{1}{2}\delta^\mu_\rho\nabla_\lambda\left(\sqrt{-g}g^{\overset{\lambda\nu}{+-}}\right) \\ +\tfrac{1}{2}\left(\sqrt{-g}g^{\mu\alpha}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\nu_\rho - \sqrt{-g}g^{\alpha\nu}\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}\delta^\mu_\rho\right) = 0. \end{aligned} \tag{5}$$

If there were no fourth bracket in (4) would imply the vanishing of $\nabla_\rho\left(\sqrt{-g}g^{\overset{\mu\nu}{+-}}\right)$, that is,

$$\nabla_\rho g^{\overset{\mu\nu}{+-}} = \frac{\partial g^{\mu\nu}}{\partial x^\rho} + g^{\alpha\nu}\Gamma^\mu_{\alpha\rho} + g^{\mu\alpha}\Gamma^\nu_{\rho\alpha} = 0. \tag{6}$$

However, this would require the vanishing of $\Gamma^\sigma_{\underset{\vee}{\alpha\sigma}}$ to which demand we have no right for the time being.

We can resolve this difficulty in the following manner. We can compute the equations of (4)

$$\begin{aligned} -\left[\tfrac{\partial}{\partial x^\rho}\left(\sqrt{-g}g^{\underline{\mu\nu}}\right) - \sqrt{-g}g^{\underline{\mu\nu}}\Gamma^\alpha_{\underline{\rho\alpha}} + \sqrt{-g}g^{\underline{\mu\alpha}}\Gamma^\nu_{\underline{\rho\alpha}} + \sqrt{-g}g^{\overset{\mu\alpha}{\vee}}\Gamma^\nu_{\underset{\vee}{\rho\alpha}}\right] + \tfrac{1}{2}\delta^\nu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\underline{\mu\lambda}}\right) + \sqrt{-g}g^{\underline{\alpha\lambda}}\Gamma^\mu_{\underline{\alpha\lambda}} + \sqrt{-g}g^{\overset{\alpha\lambda}{\vee}}\Gamma^\mu_{\underset{\vee}{\alpha\lambda}}\right] \\ +\tfrac{1}{2}\delta^\mu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\underline{\lambda\nu}}\right) + \sqrt{-g}g^{\underline{\lambda\alpha}}\Gamma^\nu_{\underline{\lambda\alpha}} + \sqrt{-g}g^{\overset{\lambda\alpha}{\vee}}\Gamma^\sigma_{\underset{\vee}{\lambda\alpha}}\right] - \left(\sqrt{-g}g^{\overset{\alpha\mu}{\vee}}\Gamma^\nu_{\underset{\vee}{\alpha\rho}} + \sqrt{-g}g^{\underline{\alpha\nu}}\Gamma^\mu_{\underline{\alpha\rho}}\right) = 0 \end{aligned}$$

and

$$\begin{aligned} -\left[\tfrac{\partial}{\partial x^\rho}\left(\sqrt{-g}g^{\overset{\mu\nu}{\vee}}\right) - \sqrt{-g}g^{\overset{\mu\nu}{\vee}}\Gamma^\alpha_{\underline{\rho\alpha}} + \sqrt{-g}g^{\underline{\mu\alpha}}\Gamma^\nu_{\underset{\vee}{\rho\alpha}} + \sqrt{-g}g^{\overset{\mu\alpha}{\vee}}\Gamma^\nu_{\underline{\rho\alpha}}\right] + \tfrac{1}{2}\delta^\nu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\overset{\mu\lambda}{\vee}}\right) + \sqrt{-g}g^{\overset{\alpha\lambda}{\vee}}\Gamma^\mu_{\underline{\alpha\lambda}} + \sqrt{-g}g^{\underline{\alpha\lambda}}\Gamma^\mu_{\underset{\vee}{\alpha\lambda}}\right] \\ +\tfrac{1}{2}\delta^\mu_\rho\left[\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\overset{\lambda\nu}{\vee}}\right) + \sqrt{-g}g^{\overset{\lambda\alpha}{\vee}}\Gamma^\nu_{\underline{\lambda\alpha}} + \sqrt{-g}g^{\underline{\lambda\alpha}}\Gamma^\sigma_{\underset{\vee}{\lambda\alpha}}\right] - \left(\sqrt{-g}g^{\underline{\alpha\nu}}\Gamma^\mu_{\underset{\vee}{\alpha\rho}} + \sqrt{-g}g^{\overset{\alpha\rho}{\vee}}\Gamma^\mu_{\underline{\alpha\rho}}\right) = 0. \end{aligned}$$

Therefore, we form of the second equation

$$\begin{aligned} -\tfrac{\partial}{\partial x^\rho}\left(\sqrt{-g}g^{\overset{\mu\nu}{\vee}}\right) - \sqrt{-g}g^{\underline{\alpha\nu}}\Gamma^\mu_{\underset{\vee}{\alpha\rho}} - \sqrt{-g}g^{\overset{\alpha\nu}{\vee}}\Gamma^\mu_{\underline{\alpha\rho}} - \sqrt{-g}g^{\underline{\mu\alpha}}\Gamma^\nu_{\underset{\vee}{\rho\alpha}} - \sqrt{-g}g^{\overset{\mu\alpha}{\vee}}\Gamma^\nu_{\underline{\rho\alpha}} \\ +\sqrt{-g}g^{\overset{\mu\nu}{\vee}}\Gamma^\alpha_{\underline{\rho\alpha}} + \tfrac{1}{2}\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\overset{\mu\lambda}{\vee}}\right)\delta^\nu_\rho + \tfrac{1}{2}\tfrac{\partial}{\partial x^\lambda}\left(\sqrt{-g}g^{\overset{\lambda\nu}{\vee}}\right)\delta^\mu_\rho = 0. \end{aligned}$$

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References

[1] M. Valenzuela. A relativistic theory of the field. II: Hamilton's principle and Bianchi's identities, Revista de Investigación de Física, **24**(**3**) 12-24 (2021). Doi: https://doi.org/10.15381/rif.v24i3.14375