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Corrigendum: On the relativistic theory of the asymmetric field

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1 Introduction

In writing out the eq. (43) for in my recent paper [1], an inadvertent error led to two errant numerical expressions in "Diferential identities". The main results and conclusions remain unchanged. All of the affected formulas are corrected below.

2 Differential Identities

We take an infinitesimal transformation defined of the form

$$x^{\mu} \longrightarrow x^{\mu *} = x^{\mu} + \xi^{\mu}, \tag{1}$$

where ξ^{μ} is an infinitesimal vector.

Under the transformation law

$$\sqrt{-g}g^{\rho\sigma *} = \frac{\partial x^{\rho *}}{\partial x^{\mu}}\frac{\partial x^{\sigma *}}{\partial x^{\nu}}\sqrt{-g}g^{\mu\nu}\left|\frac{\partial x^{\tau}}{\partial x^{\tau *}}\right|,$$

and with the help of equation (1), we obtain the variation

$$\delta\left(\sqrt{-g}g^{\mu\nu}\right) = \sqrt{-g}g^{\lambda\nu}\frac{\partial\xi^{\mu}}{\partial x^{\lambda}} + \sqrt{-g}g^{\mu\lambda}\frac{\partial\xi^{\nu}}{\partial x^{\lambda}} - \sqrt{-g}g^{\mu\nu}\frac{\partial\xi^{\lambda}}{\partial x^{\lambda}} + \left[-\frac{\partial\left(\sqrt{-g}g^{\mu\nu}\right)}{\partial x^{\lambda}}\xi^{\lambda}\right], \tag{2}$$

where $\delta\left(\sqrt{-g}g^{\mu\nu}\right) = \sqrt{-g}g^{\mu\nu *} - \sqrt{-g}g^{\mu\nu}$.

The variation for the pseudo tensor $U^{\rho}_{\mu\nu}$; with the help of equation (1) and equation (15) from [1]:

$$\delta U^{\rho}_{\eta\sigma} = U^{\rho *}_{\eta\sigma} - U^{\rho}_{\eta\sigma} = U^{\lambda}_{\eta\sigma}\frac{\partial\xi^{\rho}}{\partial x^{\lambda}} - U^{\rho}_{\lambda\sigma}\frac{\partial\xi^{\lambda}}{\partial x^{\eta}} - U^{\rho}_{\eta\lambda}\frac{\partial\xi^{\lambda}}{\partial x^{\sigma}} - \frac{\partial^2\xi^{\mu}}{\partial x^{\eta}\partial x^{\sigma}} + \left[-\frac{\partial U^{\rho}_{\eta\sigma}}{\partial x^{\lambda}}\xi^{\lambda}\right]. \tag{3}$$

In the variational calculus, the variations (2) and (3) represent the variations for fixed points of the coordinates. To obtain these, the terms in the parentheses have to be added. If these transform variations are substituted; that is, equations (2) and (3), in the integral

$$\int dt\int d^3x\left\{\delta\left(\sqrt{-g}g^{\eta\sigma}\right)S_{\eta\sigma} - \sqrt{-g}n^{\mu\nu}_{\lambda}\delta U^{\lambda}_{\mu\nu}\right\},$$

it becomes, therefore, identically null. Considering that this integral depends linearly and homogeneously on ξ^{μ} and its derivatives, it can be represented as follows

$$\int dt\int d^3x\sqrt{-g}m_{\mu}\xi^{\mu} = 0,$$

and through repeated integration by parts, the differential identities of the integrand are deduced, ($\sqrt{-g}m_{\mu} \equiv 0$), i. e.,

$$-\sqrt{-g}n^{\eta\sigma}_{\rho}\frac{\partial U^{\rho}_{\eta\sigma}}{\partial x^{\nu}} + \frac{\partial}{\partial x^{\eta}}(\sqrt{-g}n^{\lambda\sigma}_{\nu}U^{\eta}_{\lambda\sigma} - \sqrt{-g}n^{\eta\sigma}_{\rho}U^{\rho}_{\eta\nu}) + \frac{\partial}{\partial x^{\eta}}\left[-\sqrt{-g}n^{\eta\sigma}_{\rho}U^{\rho}_{\nu\sigma} - \frac{\partial\left(\sqrt{-g}n^{\eta\sigma}\right)}{\partial x^{\sigma}}\right] - \frac{\partial\mathfrak{g}^{\eta\sigma}}{\partial x^{\nu}}S_{\eta\sigma} - \frac{\partial}{\partial x^{\lambda}}\left(\mathfrak{g}^{\lambda\sigma}S_{\nu\sigma} + \mathfrak{g}^{\eta\lambda}S_{\eta\nu} - \delta^{\lambda}_{\nu}\mathfrak{g}^{\eta\sigma}S_{\eta\sigma}\right) = 0, \tag{4}$$

with $\mathfrak{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$. These are four differential identities for the first terms of the field equations (22) and (23) in [1], which are equivalent to the Bianchi identities.

There is a fifth identity corresponding to the invariance of the action integral with respect to infinitesimal λ transformations. By substituting $\delta\left(\sqrt{-g}g^{\mu\nu}\right) = 0$ and $\delta U^{\rho}_{\mu\nu} = \delta^{\rho}_{\mu}\partial_{\nu}\lambda - \delta^{\rho}_{\nu}\partial_{\mu}\lambda$ in the integral

$$\int dt\int d^3x\left\{\delta\left(\sqrt{-g}g^{\eta\sigma}\right)S_{\eta\sigma} - \sqrt{-g}n^{\mu\nu}_{\lambda}\delta U^{\lambda}_{\mu\nu}\right\},$$

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we find,

$$\int \sqrt{-g} n^{\mu\nu}_{\rho} \left(\delta^{\rho}_{\mu} \frac{\partial \lambda}{\partial x^{\nu}} - \delta^{\rho}_{\nu} \frac{\partial \lambda}{\partial x^{\mu}} \right),$$

after an integration by parts:

$$2 \int \frac{\partial}{\partial x^{\lambda}} \left(\sqrt{-g} n_{\sigma}^{\overset{\lambda\sigma}{\vee}} \right) \lambda d^4 x = 0$$

the desired identity

$$\frac{\partial \left(\sqrt{-g} n_{\sigma}^{\overset{\lambda\sigma}{\vee}} \right)}{\partial x^{\lambda}} \equiv 0. \tag{5}$$

For $\sqrt{-g} n_{\eta}^{\overset{\mu\eta}{\vee}}$, from field equations

$$S_{\eta\sigma} \equiv \frac{\partial U^{\rho}_{\eta\sigma}}{\partial x^{\rho}} - U^{\alpha}_{\eta\rho} U^{\rho}_{\alpha\sigma} + \frac{1}{3} U^{\beta}_{\eta\beta} U^{\alpha}_{\alpha\sigma} = 0, \tag{6}$$

$$\sqrt{-g} n^{\mu\nu}_{\lambda} = \frac{\partial}{\partial x^{\lambda}} \left(\sqrt{-g} g^{\mu\nu} \right) + \sqrt{-g} g^{\eta\nu} \times \left(U^{\mu}_{\eta\lambda} - \frac{1}{3} \delta^{\nu}_{\lambda} U^{\rho}_{\eta\rho} \right) + \sqrt{-g} g^{\mu\eta} \left(U^{\nu}_{\eta\lambda} - \frac{1}{3} \delta^{\mu}_{\lambda} U^{\rho}_{\eta\rho} \right), \tag{7}$$

we find

$$\sqrt{-g} n_{\eta}^{\overset{\mu\eta}{\vee}} = \frac{\partial \left(\sqrt{-g} g^{\overset{\mu\eta}{\vee}} \right)}{\partial x^{\eta}} = 0,$$

equation that expresses the nullity of the magnetic density and $g^{\overset{\mu\eta}{\vee}}$ plays the role of the electromagnetic potential vector. If we name

$$(G^{\mu} \equiv) \frac{\partial}{\partial x^{\rho}} \left(\sqrt{-g} g^{\overset{\mu\rho}{\vee}} \right) = 0 \tag{8}$$

then

$$\left(G^{\overset{\mu\nu}{\vee}} \equiv \right) \frac{\partial^2 \left(\sqrt{-g} g^{\overset{\mu\nu}{\vee}} \right)}{\partial x^{\alpha} \partial x^{\alpha}} = 0. \tag{9}$$

We now have the identity

$$\frac{\partial G^{\overset{\mu\nu}{\vee}}}{\partial x^{\nu}} - \frac{\partial^3 \left(\sqrt{-g} g^{\overset{\mu\nu}{\vee}} \right)}{\partial x^{\nu} \partial x^{\alpha} \partial x^{\alpha}} \equiv 0$$

or

$$\frac{\partial G_{\underset{\vee}{\mu\nu}}}{\partial x^{\nu}} - \frac{\partial^2 G_{\mu}}{\partial x^{\alpha} \partial x^{\alpha}} \equiv 0. \tag{10}$$

After differenciate equation (10) with to respect to ρ, we found the next expression

$$\frac{\partial G_{\underset{\vee}{\mu\nu}}}{\partial x^{\rho}} - \frac{\partial^2}{\partial x^{\alpha} \partial x^{\alpha}} \left[\frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\mu\nu}} \right)}{\partial x^{\rho}} \right] = 0. \tag{11}$$

After applying two cyclic permutations of the indices μ, ν and ρ, we obtain

$$\frac{\partial G_{\underset{\vee}{\mu\nu}}}{\partial x^{\rho}} + \frac{\partial G_{\underset{\vee}{\rho\mu}}}{\partial x^{\nu}} + \frac{\partial G_{\underset{\vee}{\nu\rho}}}{\partial x^{\mu}} - \frac{\partial^2}{\partial x^{\alpha} \partial x^{\alpha}} \times \left[\frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\mu\nu}} \right)}{\partial x^{\rho}} + \frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\rho\mu}} \right)}{\partial x^{\nu}} + \frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\nu\rho}} \right)}{\partial x^{\mu}} \right] \equiv 0. \tag{12}$$

Therefore, the equations which according to field equations hold for an antisymmetric field are

$$\frac{\partial}{\partial x^{\alpha}} \left(\sqrt{-g} g^{\overset{\mu\rho}{\vee}} \right) = 0 \tag{13}$$

$$\frac{\partial^2}{\partial x^{\alpha} \partial x^{\alpha}} \left[\frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\mu\nu}} \right)}{\partial x^{\rho}} + \frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\rho\mu}} \right)}{\partial x^{\nu}} + \frac{\partial \left(\sqrt{-g} g_{\underset{\vee}{\nu\rho}} \right)}{\partial x^{\mu}} \right] \equiv 0. \tag{14}$$

If, in the equation (14), the expression inside the parentheses would itself vanish, then we would have Maxwell's equations for empty space.

If this is taken, the expression

$$\frac{\partial g_{\underset{\vee}{\mu\nu}}}{\partial x^{\eta}} + \frac{\partial g_{\underset{\vee}{\nu\eta}}}{\partial x^{\mu}} + \frac{\partial g_{\underset{\vee}{\eta\mu}}}{\partial x^{\nu}} = 0, \tag{15}$$

expresses the current density. Furthermore, the divergence of this magnitude becomes identically zero.

The system (15) thus contains essentially four equations which are written out as follows:

$$\frac{\partial g_{\underset{\vee}{23}}}{\partial x^{0}} + \frac{\partial g_{\underset{\vee}{30}}}{\partial x^{2}} + \frac{\partial g_{\underset{\vee}{02}}}{\partial x^{3}} = 0, \tag{16}$$

$$\frac{\partial g_{\underset{\vee}{30}}}{\partial x^{1}} + \frac{\partial g_{\underset{\vee}{01}}}{\partial x^{3}} + \frac{\partial g_{\underset{\vee}{13}}}{\partial x^{0}} = 0, \tag{17}$$

$$\frac{\partial g_{\underset{\vee}{01}}}{\partial x^{2}} + \frac{\partial g_{\underset{\vee}{12}}}{\partial x^{0}} + \frac{\partial g_{\underset{\vee}{20}}}{\partial x^{1}} = 0, \tag{18}$$

$$\frac{\partial g_{\underset{\vee}{12}}}{\partial x^{3}} + \frac{\partial g_{\underset{\vee}{23}}}{\partial x^{1}} + \frac{\partial g_{\underset{\vee}{31}}}{\partial x^{2}} = 0. \tag{19}$$

This system correspond to the second of Maxwell's system of equations. We recognize this at once by setting

$$\begin{array}{lll} g_{\underset{\vee}{23}} = H_x, & g_{\underset{\vee}{31}} = H_y, & g_{\underset{\vee}{12}} = H_z, \\ g_{\underset{\vee}{10}} = E_x, & g_{\underset{\vee}{20}} = E_y, & g_{\underset{\vee}{30}} = E_z. \end{array} \tag{20}$$

Then in place of (16), (17), (18) and (19) we may set, in the usual notation of the three-dimensional vector analysis

$$-\frac{\partial \overrightarrow{H}}{\partial t} = \nabla \times \overrightarrow{E}, \tag{21}$$

$$\nabla \cdot \overrightarrow{H} = 0. \tag{22}$$

Now, we take $0 = \frac{\partial}{\partial x^{\nu}} g_{\underset{\vee}{\mu\nu}}$ we obtain

$$\nabla \cdot \overrightarrow{E} = 0, \tag{23}$$

$$\nabla \times \overrightarrow{H} = \frac{\partial \overrightarrow{E}}{\partial t}. \tag{24}$$

Therefore, we deduce the Maxwel's first system. Thus (13) and (14) are substantially the Maxwell's equations of empty space.

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References

[1] M. Valenzuela, On the relativistic theory of assymetric field, Revista de Investigación de Física, **25**(**2**) (2022) 20-34. Doi: https://doi.org/10.15381/rif.v25i2.21449