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SEARCHING FOR REGULARITIES IN THE SQUARE NET SITE PERCOLATION

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Abstract

A statistical study of nearest neighbors site percolation in 75x75, 150x150 and 225x225 square lattices has been performed with the aim on finding out possible regularities; to accomplish this, a percolation Monte Carlo simulation software was specially developed and statistics from 100 completed percolations in each lattice were obtained.

The investigation was carried-out on the threshold of percolation, this is, setting the probability in the critical value P = Pc = 0.5928 throughout all the simulations. It is important to notice the fact that in order to achieve the 100 successful percolations required for the statistics, it was necessary to execute the simulations many more times, because very many of them resulted frustrated, not attaining percolation. Another important observation is the fact that percolation was not always achieved by the largest clusters, there were instances where the percolating clusters were unsuspected thin clusters far from being the fattest ones.

Regularities have been observed in the number of resulting clusters and in the number of step necessary to achieve percolation. It is concluded that additional research with much larger lattices would help to wipe away some subsisting doubts.

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Keywords: Percolation, Infiltration, Monte Carlo, Computational, Simulational, Physics, Clusters.

Resumen

Con la finalidad de identificar posibles regularidades, se ha realizado un estudio estadístico de percolación entre los vecinos más próximos, en redes cuadradas de 75x75, 150x150 y 225x225. Para este fin se ha desarrollado especialmente un software de simulación Monte Carlo, con el que se han obtenido datos de 100 percolaciones completas para cada red.

La investigación se ha llevado a cabo en el umbral de percolación, es decir, fijando la probabilidad durante toda la investigación, en el valor critico P = Pc = 0.5978 Es importante mencionar que para lograr las 100 percolaciones completas necesarias para las estadísticas, fue necesario ejecutar muchas mas simulaciones, ya que muchas de ellas resultaron frustradas, no logrando percolación completa. Otra observación importante es el hecho de que la percolación no es necesariamente lograda siempre por el racimo más grande, se detecto muchos casos en fue lograda por racimos insospechados, dada su poca población.

Se detectaron regularidades en el número de racimos resultantes y en el número de pasos necesarios para lograr la percolación. Se concluye en que una investigación usando redes mucho mayores ayudaría a despejar algunas dudas que aun subsisten.

Palabras claves: Percolación, Infiltración, Monte Carlo, Física, Computacional, Simulacional, Racimos.

1. Introducción

The word "Percolation" comes from the Latin "Per" meaning "through" and "Colare", meaning "to flow" [5]. Percolation deals with the structural change from a collection of many disconnected elements into a single large cluster of connected elements. The most popular example of percolation is that of the hot water flowing through some coffee

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grind and dripping into the coffeepot as an aromatic and delicious coffee.

Percolation [1-4] also known as Infiltration, is used to study the electrical properties in disordered systems like amorphous semiconductors and crystalline semiconductors with impurities. An example of an application of percolation is the formation of thin gold films on an amorphous substrate, at the percolation point (the percolation threshold) the film provides electrical conductivity. Percolation knowledge has also been applied to the secondary recovery of petroleum.

Percolation belongs to Critical Phenomena because in the Critical Point some properties of the system change abruptly. When the probability P of the control parameter reaches a value Pc, the system suddenly changes from a collection of very many disconnected elements into a large conglomerate, this is a Change of Phase is produced.

Percolation may also be used in social sciences to study the propagation of rumors and may capitalized by ill-intentioned politicians. Percolation simulations have also been used to study epidemic propagations.

The minimum value of the probability P for percolation to take place is the Percolation Threshold Pc, in the case of bi-dimensional nearest neighbor site-percolation on the square lattice the Critical value of P is Pc = 0.5928 , [1] and this value gives the position of a Phase Transition without broken symmetry [1].

The incipient infinite cluster, this is simply, the percolating cluster, is statistically self similar, has a fractal structure and has a fractal dimension [6].

Even in the simplest case, that of the two-dimensional square lattice site percolation, does not exist nowadays an exact solution of the percolation problem, and no exact results are known on any kind of lattice in three dimensions [7].

Typical lattice size for percolation systems is about a million sites [7,8], these simulations are usually executed on super computers. The work being reported here, was carried out in a simple PC where the largest lattice was 225x225 = 50625 sites, efforts are being made to achieve larger lattice sizes.

2. Description of the simulation software.

Under the sponsorship of VirtualDynamics the author of this investigation devised and developed a Percolation Monte Carlo simulation software, having the following features:

The software simulates Nearest-Neighbors Site Percolation in at most a 230x230 square lattice and in order to make this software intuitively-easy-to-use, the forest-fire paradigm [1] in a square-shaped forest of trees is made, thus the 230x230 square matrix used for the simulations may lodge at most 52900 planted trees, one per site. The forest used in this simulator is not natural, it is rather artificial, because trees are planted in rows and columns.

Initially green trees are randomly distributed on the forest with a Probability P, (concentration of living trees, P has values from 0 to 1), independent of its neighbors. There are also empty (black) sites with Probability (concentration) 1-P.

Once trees have been planted on the forest, all trees on the top row of the matrix get fire (become red), this spreads from any red tree to any nearest neighboring green tree. Percolation takes place as soon as fire traverses the forest from top to bottom row.

During simulation the lattice is swept from top to bottom, each sweep through the whole lattice constitutes one time unit. In a given sweep an ignited (red) tree passes fire to its nearest neighboring green trees, the latter becomes red and the former becomes Yellow (begins to quench) and it can not ignite any other tree any more, finally this yellow tree becomes white (completely quenched).

The shortest duration of the fire traversing the forest would be equal to its length, and this would occur when the probability is P = 1, because in this case every simulation step would carry the fire one step forward.

As fire is propagated from one tree to its neighbors and some burning trees become quenched, and thus unable to spread the fire, clusters of fire-propagation trees are formed.

The software has been prepared so that once percolation is achieved, every developed cluster is individualized, and its population counted. After fire percolates the forest, the simulator displays the clusters with different colors so that they can be easily visualized. The forest fire Lifetime is given by the number of sweeps through the forest until percolation takes place. The simulator is also enabled to identify the largest (most populated) and the percolating clusters.

It can easily be seen that Percolation depends only in the Probability P, no matter either the dimensions of the forest or the random seed.

3. Experimental Results

Maintaining fixed the probability throughout all the investigation in Pc = 0.5928, statistics from 100 successful percolations in three different lattice-types, 75x75, 150x150 and 225x225, were computed.

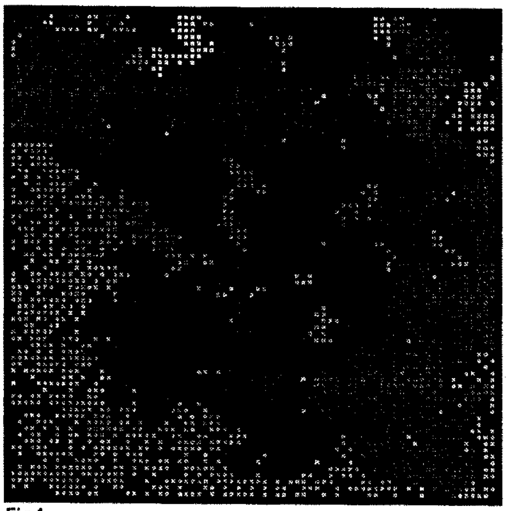


Fig 1

Fig. 1.- Shows the 21 clusters resulting in a 75x75 lattice simulation, it can be seen that in this particular case percolation was not attained by the largest cluster, which has a population of 37.68%, but by a smaller cluster having a population of 20.32%

Table (1) Statistics resulting from 100 achieved percolations using three different lattices: 75x75, 150x150 and 225x225

Lattice size		75x75	150x150	225x225
Number of Simulations to achieve 100 percolations		205	190	186
Planted Trees (100%)	Min	3246	13190	29846
	Ave	3337	13352	30038
	Max	3421	13603	30294
Fire Duration	Min	99	216	348
	Ave	124	270	434
	Max	204	359	701
Developed Clusters	Min	13	30	46
	Ave	18	36	55
	Max	22	45	64
Burnt trees %	Min	28.355	26.446	30.706
	Ave	57.603	52.688	51.704
	Max	75.595	73.586	70.779
Max Cluster Pop %	Min	16.934	16.599	15.725
	Ave	34.137	31.098	30.932
	Max	57.300	53.648	48.742
Percolating Cluster Pop %	Min	16.934	12.176	13.111
	Ave	33.535	30.464	30.697
	Max	57.300	53.648	48.742
Duration / Percolating Pop	Min	0.061	0.033	0.027
	Ave	0.118	0.072	0.050
	Max	0.221	0.166	0.101
Percolations not achieved by largest clusters		7	7	6

Seven graphs have been generated, they show respectively:

(1) The number of randomly planted trees for probability P = 0.5928
(2) The fire lifetimes, the fire durations for percolations to be completed.
(3) The number of developed clusters.
(4) The percentage of burnt trees.
(5) The population percentages of the largest (most populated) clusters.
(6) The population percentages of the clusters achieving percolation.
(7) The fraction given by Fire Duration divided by Percolating Population.

The following observations are based on table (1) and on the seven resulting graphs. In table (1), Min, Max and Ave refer to the minimum, the maximum and the average respectively of the 100 collected data for each case.

Number of simulations to get 100 Percolations.-
In order to collect data from 100 successful experiments in the three cases being studied, more than 100 executions were necessary in each case. It can be seen that the larger the simulation matrix, the smaller the number of required trials to achieve 100 percolations, though apparently this number tends to stabilize around 190. This point is not very important, but it may require more specific research if the need to know more about arises.

Number of randomly planted trees.-
In the three cases, the average of randomly planted trees is in agreement with the probability (P = Pc = 0.5928) used in this research. In the 75x75 case, the 3337 trees are the 59.32% of the 100% given by 5625. When the lattice is 150x150 the 100% would be 22500 and the 13352 trees are just the 59.3%. Same situation for the 225x225 case.

Fire lifetime, Fire duration.-
As expected, the results show that the larger the lattice, the longer the fire duration. It can be seen in Graph 2 that the dispersion of the fire life times from the averages is larger for larger lattices, this may be explained by the fact that the number of developed clusters (see Graph 3) has also larger fluctuations the larger the lattice size, this is explained by the fact that with larger lattices there are more possibilities of finding trees to get fire.

Burnt trees percentage.-
Results were so dispersed (see Graph 4) that no regularities were observed.

Largest clusters populations.-
Results were so dispersed (see Graph 5) that no regularities were observed.

Percolating clusters populations.-

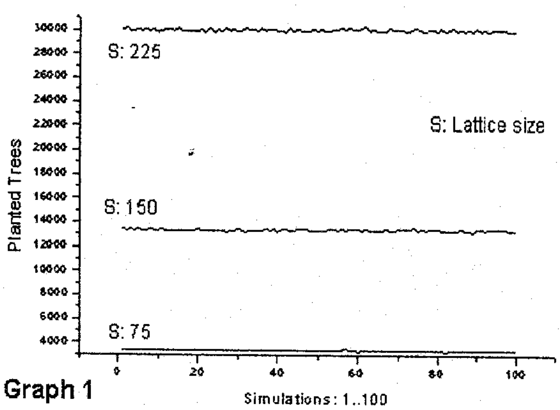
Results were so dispersed (see Graph 6) that no regularities were observed, at a glance Graph 6 is equal to Graph 5, but actually they are different, though only slightly, comparing in table (1) results for max cluster population with those for percolating cluster population, it is seen that results are very close.

Relation of Fire duration with Percolating population.-

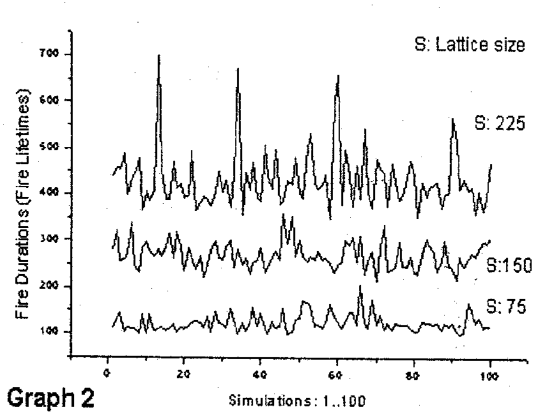
Since fire lifetime and percolating population are both random-valued results from every simulation, this investigator wanted to see if there is any interesting outcome when dividing their values (see Graph 7), it seems there are traces of regularity though these are not so evident with the lattice sizes used in this research.

Percolation is not always achieved by largest clusters.-

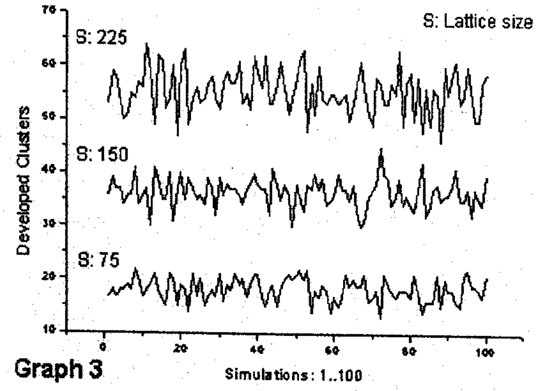
This may be appreciated in the last row of table 1, which shows the number (the percentage) of cases where percolation was not attained by the largest clusters.



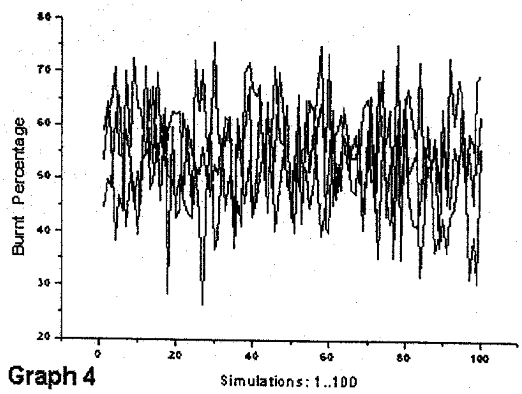
Graph 1



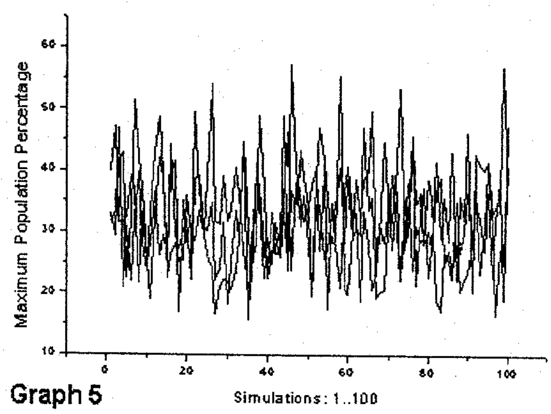
Graph 2



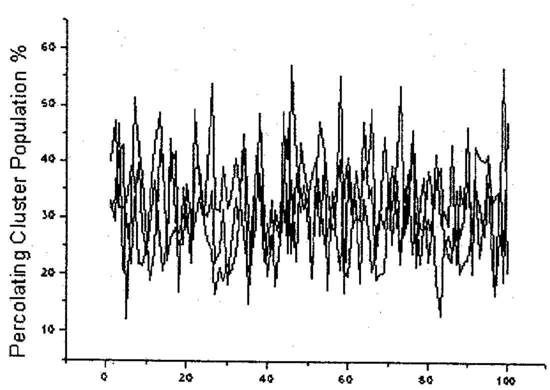
Graph 3



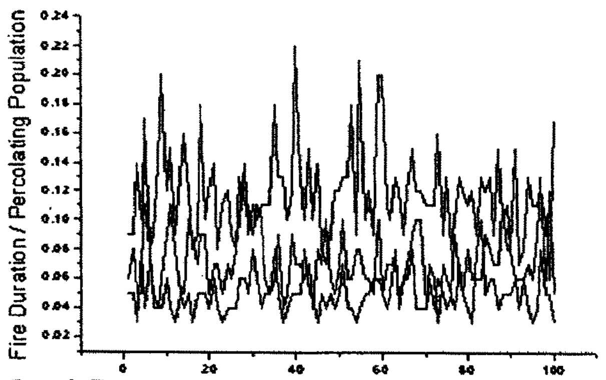
Graph 4



Graph 5



Graph 6



Graph 7 Simulations: 1..100

4. Conclusión

In this research the size of the simulating matrix, initially 75x75, and increased in steps of 75, has been the main variable, regularities for fire duration and number of developed clusters have been immediately encountered, however no regularities have been detected for the other outcomes, it may be interesting to investigate what happens when starting with let's say 75x75 and then duplicating the matrix size some 5 or 6 times, which may be achieved using much more powerful computers.

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