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Diversity and Temporality of Chaotic Events

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ABSTRACT

Publications dealing with chaos usually exhibit an image of a single, truncated and always expanding chaotic event in the system in which chaos is being reported. This generates the impression that chaotic events are unique and once they start, they increase intensity ad infinitum, eventually taking control of the system, and lasting forever. With the aim on finding out whether the above described panorama is correct, an investigation was carried out on the nonlinear damped and forced oscillator (NLDFO). It was encountered a diversity of chaotic events and that those have a beginning and an end, this is, they are temporal. Additionally it has been observed that chaotic events initially generate a series of period bifurcations increasing their intensity and then by collapsing bifurcations this intensity gradually decreases until chaos vanishes. The largest Lyapunov exponents of the displayed chaotic events, as well as some general observations about chaotic events in the NLDFO are reported.

Keywords: nonlinear oscillations, chaos, numerical, simulation, runge kutta

DIVERSIDAD Y TEMPORALIDAD DE LOS EVENTOS CAÓTICOS

RESUMEN

Las publicaciones sobre caos generalmente exhiben una imagen de un único evento caótico, truncado y siempre en expansión, del sistema en el que el caos se está divulgando. Esto genera la impresión de que los eventos caóticos son únicos y una vez que comienzan, aumentan su intensidad ad infinitum, tomando eventualmente el control del sistema y permaneciendo para siempre. Con el objetivo de averiguar si es exacto el panorama descrito, se llevó a cabo una investigación con el oscilador no lineal amortiguado y forzado (ONLAF). Se encontró una diversidad de eventos caóticos y, que los mismos tienen un inicio y un final, es decir, son temporales. Se descubrió además que los eventos caóticos generan inicialmente una serie de bifurcaciones del periodo, incrementando su intensidad y, luego mediante colapsos de las mencionadas bifurcaciones, su intensidad disminuye gradualmente, hasta que el caos desaparece. Se reportan los mayores exponentes de Lyapunov de los eventos caóticos mostrados, así como algunas observaciones generales sobre los eventos caóticos en el ONLAF.

Palabras clave: oscilaciones no lineales, caos, simulación numérica, runge kutta

1. INTRODUCTION

1.1. Chaos is becoming ubiquitous

Industrial machinery makes ample use of rotators and oscillating gearings which, due to the combined frequencies involved in their functioning are strong candidates to experiment chaos. The dynamics of fluid mixing, which is present in the preparation of colorants, tints, paintings and foods may be understood from the point of view of chaos. Industrial machinery is controlled by complex electronic circuits prone to chaos. In economics, the dollar exchange rate in third world countries is chaotic. In physics, chaos is encountered in the nonlinear dynamics of some oscillators, and in astrophysics. In fluid dynamics engineering, chaos may help to model turbulent fluid dynamics. The muscular fibers of the heart may be regarded as a set of harmoniously pulsating oscillators, and the cardiac fibrillation preceding a cardiac collapse may be considered as a transition to chaos of the heart. Since long ago it is known that certain chemical reactions oscillate and now traces of chaotic behavior in some of those reactions have been identified.

The above mentioned examples lead to the conclusion that Chaos is becoming ubiquitous in diverse fields of everyday life.

1.2. The question leading this research

Most publications dealing with chaos exhibit an image like the one in Fig. 1, where a truncated and unique chaotic episode, whose magnitude is apparently increasing, is displayed.

This gives the impression that chaotic events are constantly amplifying intensity. In addition, since literature never comments about the uniqueness or variety of chaotic events, readers get also the impression that these events are unique, and infinite, this is, once a chaotic event starts, it takes control of the system undergoing it and, chaos never ends.

$$y(t) = h_o + v_o\, t + \frac{1}{2} g t^2 \qquad (1)$$

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A research to find out whether the above mentioned impressions are correct or not has been carried out and the results are exposed in this paper.

2. A BRIEF INTRODUCTION TO CHAOS THEORY

Chaos is the generic name of those eventual manifestations of randomness and unpredictability in completely deterministic systems.

2.1. Deterministic System

Deterministic System is a system where an initial condition completely determines the future of the system, with no randomness at all. In other words, the future states of a deterministic system depend only on its initial conditions.

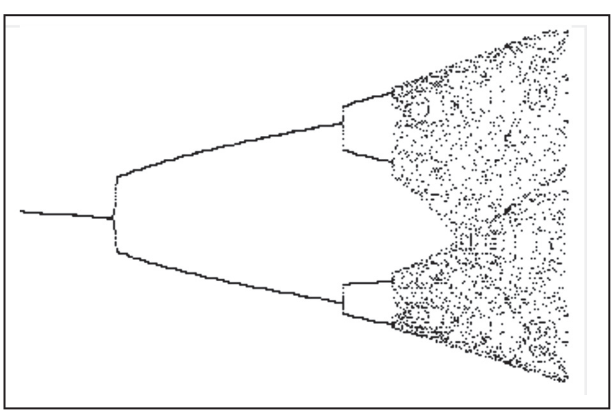
Hence, provided initial conditions are known, the future of a deterministic system can be predicted with absolute certainty. A deterministic system may be represented by a function-of-time equation.

As an example of deterministic system consider the case of an object that is launched downwards from a height ho with velocity vo1, in a place where the acceleration of the gravity is g, its position y(t) at a future time t is given by eq. (1):

The position of the object at any time t depends only on the values of ho2 vo and g

The mathematical model of a deterministic system will always return the same value at a given time, for the same initial conditions.

Fig. 1. Truncated and apparently always expanding bifurcation cascade of a chaotic event. This image produces the impression that once initiated, a chaotic event increases intensity without limit, taking control of the system and lasting forever.



2.2. Non-deterministic System

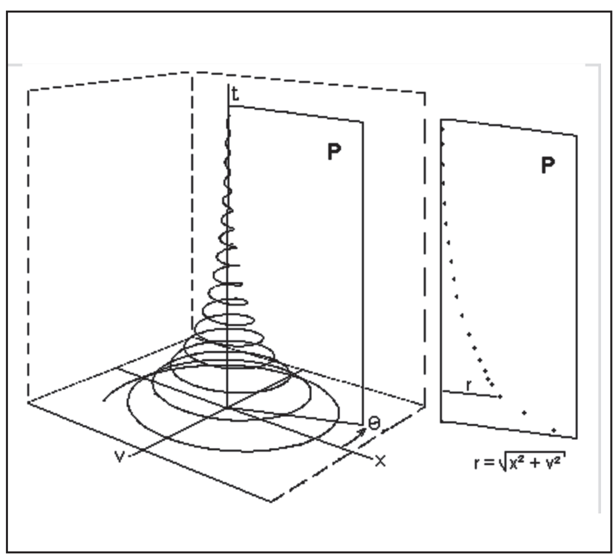
A non-deterministic system has randomness; hence a given initial condition may produce different future states. The best example of a non-deterministic system is a dice. When a dice is thrown, there is no way to predict the resulting outcome.

2.3. The Sate Space and the Poincare Map

The State Space (Fig.2) of a dynamical system is the tridimensional plotting of velocity and displacement versus time. The sequence of the (x, v, t) points as time elapses is known as "The Flow" of the system. Since time is always positive this plotting evolves only in the direction of increasing time, which is usually presented upwards.

An approach to study the evolution of a chaotic dynamical system is to analyze its State Space and, one way of achieving this is by means of bi-dimensional Poincare Maps (Fig.2).

Fig. 2. The State Space is the 3D plotting of displacement and velocity versus time (time along the vertical axis). In this case the depicted curve is that of a damped oscillator, for this reason the curve shrinks with time until the oscillator eventually stops. The sketched plane P is the Poincaré section (or plane) at an angle with the x-axis. The curve intersects the Poincaré plane at some points which constitute the Poincare Map at angle Notice that theoretically there are infinite Poincaré Maps. The sequence of (x,v,t) points on State Space is known as "The Flow" of the system.



The Poincare Plane P, may be seen as a tomographic cut along time of the State Space, see Fig. 2. This plane P, is defined at some angle with the x-axis. The Poincare Map is the set of all the intersections of the (x,v,t) curve –the flow of the system- with the plane P, at a predefined angle. In this way the Poincare Map helps to detect the structure –if there is one- of the State Space at the angle it is defined. Obviously, (see Fig. 3) the structure of extreme displacements is obtained at angles θ=0 and θ=π and the structure of extreme velocities is found at (θ= π/2 and θ=3π/2 A common oscillator has two opposite vibration amplitude extremes and normally these two amplitude extremes are symmetrical. This report presents visual evidence that these amplitude extremes are not necessarily symmetrical in the chaotic NLDF oscillator.

2.4 Declaring Chaos: The Lyapunov Exponent

The most common test to declare an event as chaotic is the calculation of its largest Lyapunov exponent, if this happens to be positive, the event is considered chaotic [1,2,3,4]. The experimental procedure –not so simple in practice- is to follow two nearby orbits in state space initially separated by a very short distance fixed by the researcher, and calculate the average logarithmic rate of deformation (contraction or separation) of the orbits along time. If it is assumed that the orbit deformation can be represented by an exponential factor, then the distance between the two orbits after a time t can be expressed as.

$$d_t = e^{\lambda t}\, d_o$$

$$\frac{d_t}{d_o} = e^{\lambda t} \quad \Rightarrow \quad \lambda = \frac{1}{t} ln\left[\frac{|d_t|}{|d_o|} \right] \qquad (2)$$

Where t is the time elapsed between d_t and d_o and λ is known as the Lyapunov exponent.

Notice that the shorter the time t between both distances , the larger the Lyapunov exponent.

In general, each chaotic episode is associated to a particular orbit in state space and, in order to calculate the average Lyapunov exponent for each chaotic event, a series of measurements to assess eq.(2) must be performed at some predefined time interval in the corresponding orbits.

$$\lambda_{max} = 11.52\ s^{-1} \quad and \qquad <\lambda> = 1.92\, s^{-1}$$

While the chaotic event shown in Fig. 6, (event N.° 11) has 1.5 million time steps,Δt=0.020 s and

$$\lambda_{max} = 36.63\ s^{-1} \quad and \quad <\lambda> = \ 22.66\, s^{-1}$$

Fig. 3. Projection of State Space (x,v,t) points over the XV-plane. The sketch shows the connection of X-V axes with different angles for Poincare Maps. Planes at θ=0 and θ= π collect the extreme values (positive and negative) of the oscillation displacement, while planes at θ=π/2 and θ=3π/2, capture the extreme values of oscillation velocity.

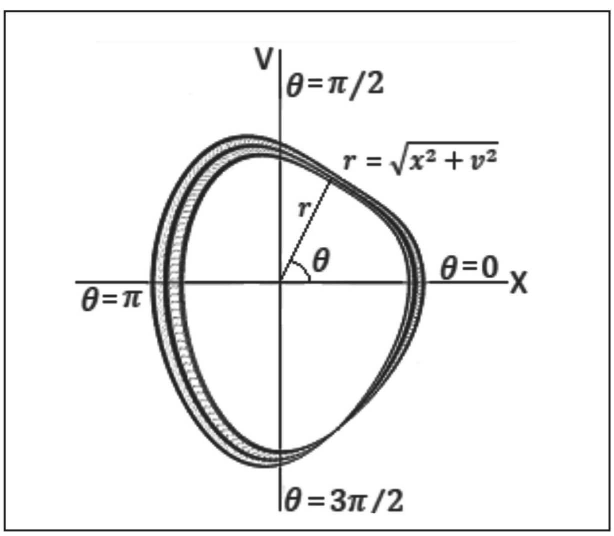
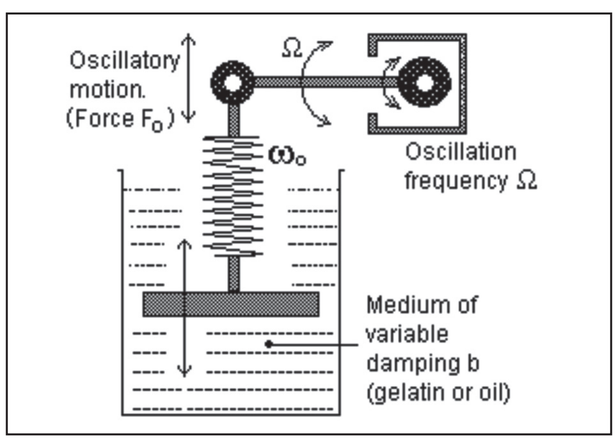


Fig. 4. Forced oscillations in a medium of variable damping. As the immersed piece oscillates in gelatin or oil, the temperature of the medium varies, changing its density and viscosity.



3. A VIRTUAL LAB TO STUDY CHAOS

A Virtual Lab (highly interactive and integrated computer program) to investigate chaos in some nonlinear systems has been completely developed from scratch by the author of this report. This Virtual Lab uses the Runge-Kutta method to numerically solve the differential equation of the nonlinear system under research. Currently this VirtualLab executes simulations up to 30 million time steps (iterations) and it generates diverse graphs. An interesting feature of the mentioned VirtualLab is that it simultaneously shows on computer screen a vibrator oscillating according to the evolution of the simulation and while the State Space is also depicted.

This means that the researcher's appreciation of chaos is not limited to interpreting terminated plottings, but many details are observed directly (in real-time) by visualizing the motion of the oscillator. This latest feature helps to understand how the intricate motion of the oscillator is depicted in state space.

4. THE MATHEMATICAL MODEL USED IN THE INVESTIGATION

The investigation here reported is based on computer simulation of the oscillating system depicted in Fig. 4, whose differential equation of motion (Eq. 3) is that of the nonlinear damped and forced (NLDF) oscillator [5,6]:

$$\frac{d^2\theta}{dt^2} + \left(\frac{b}{mL}\right)\frac{d\theta}{dt} + \omega_o^2\ Sin\ \theta = \left(\frac{F_o}{mL}\right) Sin\ \Omega t \quad (3)$$

The system is an oscillator immersed in a medium of variable damping, such as oil or gelatin (Fig.4). The continuous motion of the oscillator disturbs the temperature of the medium and in this way its density and viscosity varies while the oscillator vibrates. The oscillator, whose natural frequency is ω_o is subject to an oscillatory driving motion force F_o whose frequency is Ω .

As it can be appreciated, there are two competing frequencies in the system and there exists also an applied force and a variable damping. This constitutes the recipe [7,8] for a prone-to-chaos system. Note that the same effect may be attained without damping and with a variable force.

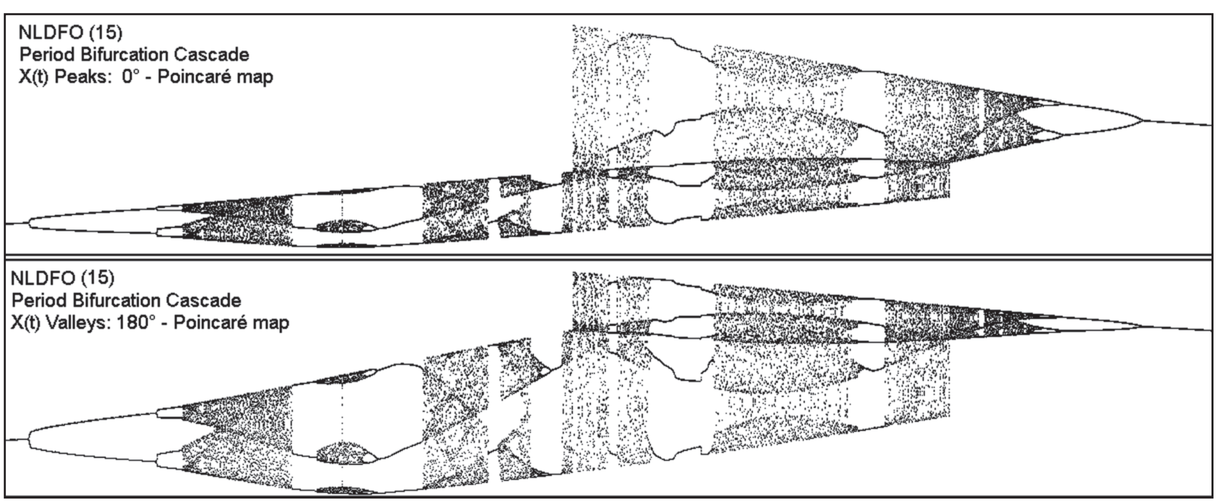
5. RESULTS OF THE INVESTIGATION

5.1. Diversity and temporality of chaotic events

The results here described may be appreciated in Figs. 5, 6 and 7.

Many isolated chaotic events were detected in the NLDF oscillator, which means that there is a diversity of chaotic events in this system [9,10], and not only a single one.

Fig. 5. Period bifurcation cascades detected in the nonlinear damped and forced oscillator (NLDFO, chaotic event 15). These plottings are Poincaré maps at θ=0 and θ= π respectively. From elementary university physics, it is known that the period of an oscillator may be calculated by measuring the distance between any two consecutive equal-phase points in its plotting of displacement versus time. In these graphs it can be seen that periods measured in the top plotting are different to those in the bottom one. It may also be appreciated that the period bifurcation cascades are not symmetrical.



It has been found that chaotic events have a beginning and an end, this is, they do not last forever. This implies that chaotic events have finiteness.

Concerning the surge of bifurcations, which may be interpreted as the intensity of the chaotic event, it was observed that initially the intensity (the number of bifurcations) is low and, little by little this intensity increases, but after some time the intensity begins to reduce –bifurcations gradually collapse- until finally the chaotic event vanishes and the system returns to its forced chaos-free oscillations regime.

In Figs. 5, 6 and 7 it can be seen that the transition towards chaos is as smooth as the transition out of it. It has been observed that the system experiencing a chaotic event terminates the events with the same smoothness it started them.

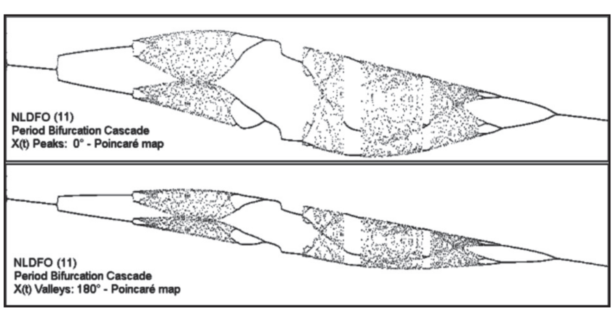
When entering a chaotic event a bifurcation of the period is observed, then each of these bifurcations bifurcates again and again, and then the system bursts into chaos. When abandoning chaos the opposite effect is observed, this is, the system collapses the period cascade until it finally winds up oscillating -free of chaos- with a single period.

5.2. Oscillation amplitudes are not symmetric

Figures 5 and 6 show two of the several chaotic events detected in the system under study. These images display the Poincare maps for 0° and for 180°, respectively. These correspond to the period bifurcation cascades observed at the mentioned angles and, making a parallel with a common oscillator, it results evident that the extremes of the amplitude oscillations are far from being symmetrical at both sides of the equilibrium position of the chaotic oscillator. In a regular oscillator the oscillation amplitudes are symmetrical about the equilibrium position of the oscillator.

Figure 7 displays period bifurcation cascades (Poincare maps at θ=0) for some additional chaotic events detected in the nonlinear damped and forced oscillator. These images make evident the diversity of chaotic events in the NLDF Oscillator.

Fig. 6. Another period bifurcation cascade detected in the NLDFO (case 11). Top: Detection with Poincaré plane at 0°. Bottom: detection with a Poincaré plane at 180°



5.3. The Lyapunov exponents

It has been encountered in this research that chaotic events are finite, then the Lyapunov exponents were calculated with eq.(2) for the complete events, this was made following Sprott's recommendation [3] of rescaling the initial values of the two state space orbits after each time-iteration.

This research has uncovered a diversity of chaotic events, and each chaotic event has its own state space, then every chaotic event must have its own Lyapunov exponent. In fact, this researcher has encountered different values of the Lyapunov exponent for different chaotic events in the NLDFO.

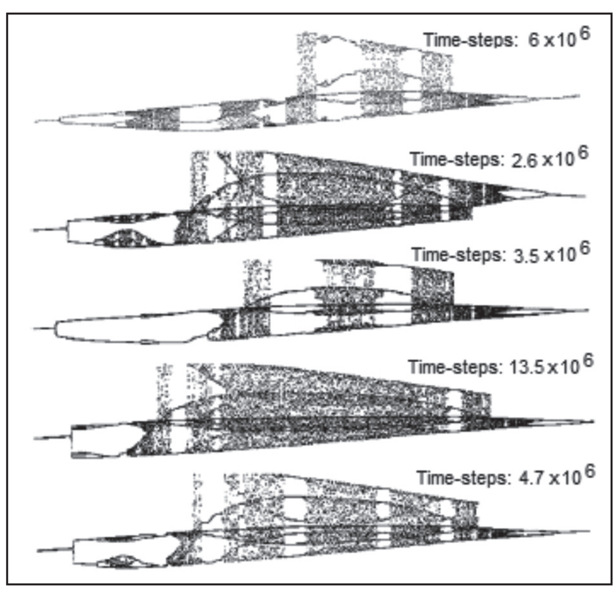
In this research an initial separation of $d_o = 10^{-8}$ between the original orbit and the test orbit in state space was used to calculate the maximum Lyapunov exponents λ.

The chaotic event show in Fig. 5, (event N.° 15), has 6 million time-steps, Δt=0.025 s and

5.4 Decreasing damping implies increasing chaos

It was experimentally discovered that as the damping of the system is little by little decreased during a chaotic event, there appeared more and more

Fig. 7. Poincaré maps at 0° for five of the many different chaotic events detected in the NLDF oscillator. All the events have a beginning and an end, they are finite. The system abandons chaos as smoothly as it entered there. In each case the number of simulation time-steps is shown.



bursts of period bifurcations. This is, as the damping was gradually lowered, new bursts of bifurcations appeared where there was no bifurcation before. This is understandable bearing in mind that more freedom (less damping) allows the system to reveal more of its chaotic character; a higher damping means more restriction to behave chaotically. These observations suggest a way to control chaos.

5.5. Chaos after chaos – Serial chaos

In order to find out what happens once a chaotic event concludes, an investigation named Chaos after chaos [11] was executed by this researcher. To achieve this investigation, the simulation time after a chaotic event finished was considerably extended (extra time), but nothing unexpected was encountered once a chaotic event finished. This seems to discard the existence of serial chaos, at least in the system under study. Bear in mind that all this is not definitive, because it may happen that a much longer extra time was needed in order to detect some interesting behavior.

In general the above described behavior sheds light on the expected comportment in other systems susceptible to undergo chaos by a period bifurcation cascade. It is opportune to mention here that not all systems prone to chaos experiment it by a bifurcation cascade, this is the case of the Duffing equation, where simulations carried out by this researcher have never generated a period bifurcation cascade.

6. CONCLUSIONS

Chaos in the nonlinear damped and forced oscillator (NLDFO) has been studied in a virtual lab expressly developed to solve its differential equation by means of the Runge-Kutta algorithm. It has been encountered that:

1. There is a variety of chaotic occurrences, this is, chaos may occur very many times.
2. Chaotic events do not last forever, they are finite.
3. The intensity of a chaotic event initially increases but after some time it begins to decrease until chaos vanishes.
4. The system abandons chaos with the same smoothness it starts that.
5. When the system transitions towards chaos there is a period bifurcation cascade and, when the system goes out of chaos, there is a collapse of cascades.
6. Once the system leaves a chaotic event, the system returns to its chaos-free forced oscillators.
7. During a chaotic event the maximum amplitudes of oscillation are far from being symmetrical about the equilibrium position of the oscillator.
8. Once a chaotic event has been detected, a decreasing damping triggers more chaos.
9. It seems to be chaos does not occur in series.

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