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Machinery Breakdown: a Chaos-Based Explanation

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ABSTRACT
An attempt to explain the process of machinery breakdown by means of Chaos Theory is presented. Machines carry out repetitive tasks that may be mathematically modelled as combinations of forced oscillatory motions; consequently these are perfect candidates to eventually evolve towards chaos. The exposition is made having as a model the route to chaos in the damped and forced non-linear oscillator.

Keywords: Route, chaos, machinery, failure, oscillator, modelling, simulation.

Rotura de maquinaria: Una explicación a partir de la Teoría del Caos

RESUMEN
Se expone una tentativa para explicar el proceso de las fallas en maquinarias, basada en la teoría del caos. Las máquinas realizan tareas repetitivas que podrían ser modeladas matemáticamente como combinaciones de movimientos oscilatorios forzados; en consecuencia éstas son perfectas candidatas a eventualmente evolucionar hacia el caos. Se hace la exposición teniendo como modelo, la ruta hacia el caos seguida por un oscilador no lineal amortiguado y forzado.

Palabras clave: Caos, ruta al caos, maquinarias, fallas, oscilador, modelación, simulación.

INTRODUCTION

Machinery used in a factory execute mainly repetitive tasks, hence its motion has a characteristic frequency and period; when a machine operates far from its characteristic frequency, it is very likely damaged and its needs reparation or maybe even replacement.

Very simple machinery, like the needle of a sewing machine has only a single frequency which takes it up and down, however more complex machinery have several interacting frequencies. As an example consider the mixer with a rotative vessel used in bakery. The helix spins with a certain frequency and the vessel rotates with another frequency.

While studying the transition to chaos in the damped non-linear forced oscillator, this author observed that its route to chaos resembles the steps followed by a machine when it is on the verge of collapsing, and that chaos, this is the absence of a frequency, when the period becomes infinite[1-3], may be paralleled to machine collapse.

In the next section the equation used by this author in the computer simulation of the transition to chaos is deduced, this may be found in any General Physics book used by university students of Physics and Engineering.

EQUATION OF MOTION

In order to construct the equation of motion for a damped non-linear forced oscillator (think about a pendulum of mass m and length L), let's remember that according to Newton's second law, the force F acting on a particle of mass m, gives this mass an acceleration, this is mathematically represented as

$$F = m\,a = m\,\frac{d^2 x}{dt^2}$$

where the acceleration of the mass is the second time derivative of its displacement x.

When F is an oscillatory sinusoidal force, like $F = -(mg/L)\mathrm{Sin}\ x$

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the equation above becomes

$$\frac{d^2 x}{dt^2} = -\,\omega_0^2\, Sin\, x$$

where the constant terms have been incorporated in ω_0, which is the (natural) frequency of the oscillator. In the case of very small oscillations (less than 15º), the oscillator executes a Simple Harmonic Motion.

If additionally the oscillating mass were immersed in a viscous medium with damping constant b, the oscillator would experiment a force opposed to its velocity, and the equation above would become

$$\frac{d^2 x}{dt^2} = -\,\omega_0^2\, Sin\, x - b\frac{dx}{dt}$$

where the first time derivative of the displacement x, is the velocity and the minus sign means that the damping force is against the direction of motion.

When the oscillator experiments an external (applied) oscillating force Fo, whose frequency is Ω, the motion is modelled by

$$\frac{d^2 x}{dt^2} = -\,\omega_0^2\, Sin\, x - b\frac{dx}{dt} + Fo\, Sin\, \Omega\, t \quad (1)$$

which is the equation of motion of a non-linear damped and forced oscillator.

Equation (1) which is a second-order differential equation, models a non-linear oscillator[2,3] (a pendulum in the simplest case) immersed in a dissipative medium of damping factor b and which is connected to an external applied sinusoidal force Fo oscillating with frequency Ω, being ω_0 the angular frequency of the free linear oscillator (without damping).

It is expected that after a transient stage the oscillator will be forced to oscillate with the frequency Ω of the applied external force.

In view of the fact that the system under investigation, represented by eq. (1) is subject to dissipation (damping b) and since there are two competing frequencies, it is expected to observe a transition to chaos[2-3].

Just to visualize equation (1) and to have an idea of the kind of motion this equation represents, in Fig. 1 a machine forcing a horizontal slab to oscillate in a medium of variable damping b is shown. The oscillating force frequency is Ω and ω_0 is the angular frequency of the free linear oscillator. It is assumed in Fig. 1 that the rotor produces an oscillatory motion governed by a Sine (or Cosine) function. A variable damping medium is one that either becomes thicker or thinner with time.

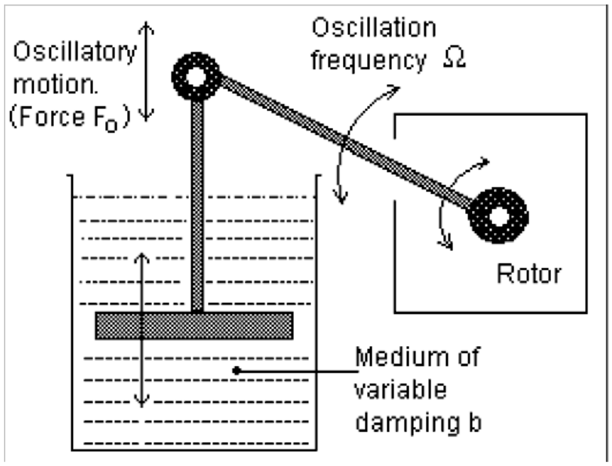

Fig. 1. Shows an object oscillating in a variable damping medium

A REAL-LIFE EXAMPLE OF A MACHINE WITH THE INGREDIENTS TO EVENTUALLY BECOME CHAOTIC

An example of an industrial system with an external applied force, competing frequencies, and a variable damping is the mixing machine used in a bakery to prepare the blend to make bread. The arm that while rotating combines the flour, butter, milk, water and other additives, turns about a vertical axis with its own frequency, the vessel containing the ingredients also rotates with a particular frequency. As time goes by, the blend becomes thicker and thicker, which is tantamount to a variable (increasing) damping.

BIFURCATION CASCADE TOWARDS CHAOS

After numerically solving equation (1) by means of the Runge-Kutta method, this author obtained the plotting shown in Fig 2. This graph displays the peaks of the oscillation amplitudes as time evolves. In region A the system oscillates with almost constant amplitude, which is associated to a single frequency. The amplitude is increasing due to the variable damping. In region B, the oscillations alternate between two amplitudes, this means that the system is oscillating with two alternating frequencies. In region C the two frequencies of region B, bifurcate each and the system oscillates with four alternating frequencies. As a whole, Fig. 2 very clearly displays the transition to chaos through a cascade of frequency bifurcations; a characteristic route taken by many systems evolving towards chaos[2].

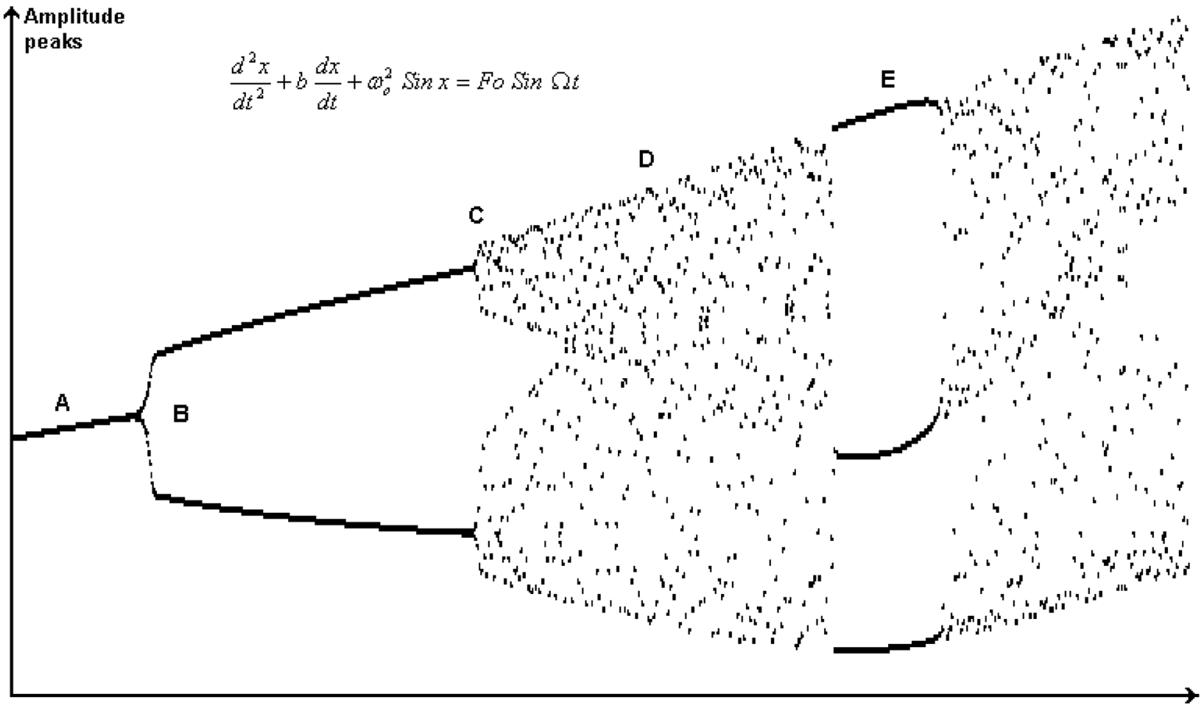

Fig2. Bifurcation cascade obtained by the author of this report by numerically solving eq. (1)

MACHINE EVOLUTION TO FAILURE

When a mechanism works well, this is without any problem at all, it shows only one type of behavior, the one it has been build for by its constructor. If a well-behaving machine produces a sound, this is a characteristic one, and the persons around the equipment know by the buzz it generates that it is operating as expected, this is represented by the almost horizontal line A (single behavior) at the left side of Fig 2. The line in region A is not strictly horizontal, because the damping is being increased.

In real life it is observed that usually when a machine is going to collapse, it starts showing a dual behavior (a behavior bifurcation), alternating between two different types of conduct, this is shown in region B in Fig 2. As the mechanism evolves towards complete disaster it shows even more different behaviors, like in region C, where every bifurcation of region B bifurcates again. Finally in region D the machine works in a complete disordered fashion (absolute chaos). At this stage in an industrial plant, the engineers have already been called or the machinery has been replaced.

A few mechanical systems can physically go beyond region C (Fig 2). Since they have not been constructed to operate with several behaviors, they simply collapse.

Sometimes however, a mechanism overcoming region D (in Fig. 2) seems to work in a new fashion, showing a not so disordered behavior, but alternating between a few conducts, like in region E, but this situation does not last for ever, eventually the system returns again to anarchy and never recovers its original behavior, roughly represented by region A

Machinery in real life have not been observed going through the cascades of bifurcations depicted in regions C and D of Fig. 2, because any machinery behaving in a weird mode is stopped and repaired or replaced. Usually machinery is not allowed to go beyond region B.

In a factory, if a machine operates in only two different modes like in region B (Fig 2), the economical consequences would be disastrous and engineers take care that machinery work in only the mode it is supposed to work (region A in Fig. 2), for this reason the kind of behavior shown in Fig. 2 is rather observed in equipment and apparatus at home or in places where a machinery misbehavior does not reflect economical losses or where it is not easy to replace.

CHAOS OF THE HEART

In the case of a healthy heart all its cells oscillate in a synchronized way. It is believed that ventricular fibrillation which precedes a heart attack, is just a period doubling cascade[4], which means that cells oscillate no more in synchronized mode.

MAN BEHAVIOR EVOLUTION TO DISASTER

Since the reaction to everyday stimuli observed in people is not always proportional to the intensity of the stimuli, it can be concluded that people's behaviour is not linear. Additionally, the stress that people endure every day is perceived like a damping on their lives and this has a cumulative effect, hence it is continually increasing. Next an attempt to explain some behaviour disorder in people is exposed.

As an adaptation of the behavior shown in Fig. 2, to human conduct, consider a man who works in an office. Normally he behaves like in region A of Fig. 2. One day he starts to alternate between two different responses to the normal stimuli in the office, this is, he shows two different conducts. Persons close to this man know that he has started drinking too much. Since nobody around the man stopped him, one day he starts showing more different behaviors (he may be in regions C and D). However after this man receives a reprimand from his superiors, he gives the impression of having improved, he is in region E. Notwithstanding, eventually this man adopts the chaotic behavior beyond E in Fig. 2, and he must be fired from work.

Psychology reports cases of persons with multiple personalities. Perhaps at the beginning, a single personality bifurcated generating a double personality and later, these also bifurcated and so on.

CONCLUSIÓN

An attempt to explain machinery breakdown by means of Chaos Theory has been presented.
This is based on the similitude in the route to chaos by means of a bifurcation cascade in a non-linear oscillator and the behavior observed in machinery when this is on the verge of collapse. An adaptation, attempting to explain human failure at the work place has also been presented.

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