PESQUIMAT, Revista de la F.C.M. de l
Universidad Nacional Mayor de San Marco
Vol. II - N°1 Pgs. 23-32 Lima - Perú Ag. 1999

# N-DIMENSIONS LINEAR THERMOELASTIC SYSTEM IN DOMAINS WITH MOVING BOUNDARY

*Cruz S. Q. De Caldas**,
*Juan Limaco,*
*Rioco K. Barreto*

## ABSTRACT

In this article the authors investigated the existence of solutions for the linear thermoelastic system in a noncylindrical domain $\hat{Q}$ of $\Re^{n+1}$.

## 1. Introduction

Let $K:[0,T]\rightarrow\Re$ be a $C^2$ function and let $\Omega$ be an open bounded set in $\Re^n$, with regular boundary. Let us consider the family of open sets in $\Re^{n+1}$:

$$\Omega_t=\{K(t)y;\quad y\in\Omega\},\quad 0\leq t\leq T,$$

with lateral boundary $\Gamma_t$. We define the noncylindrical domain $\hat{Q}$ in $\Re^{n+1}$ by:

$$Q=\bigcup_{0<t<T}\Omega_t\times\{t\},$$

with lateral boundary

$$\Sigma=\bigcup_{0<t<T}\Gamma_t\times\{t\}.$$

In this work we investigate the existence of solutions for the following thermoelastic system in $\hat{Q}$:

$$\left|\begin{array}{l}u''-\mu\Delta u-(\lambda+\mu)\nabla\, div\ u+\alpha\nabla\theta=0\quad in\ \hat{Q}\\ \theta'-\Delta\theta+\beta\ div\ u_t=0\quad in\quad \hat{Q}\\ u=0,\theta=0\ \ in\ \hat{\Sigma}\\ u(x,0)=u_0(x), u'(x,0)=u_1(x),\ \theta(x,0)=\theta_0(x)\ in\ \Omega_0.\end{array}\right.\qquad (*)$$

* Departamento de Matemática Aplicada – IMUFF, Universidade Federal Fluminense, Rua Mário Santos Braga s/n°, 24020-140 Niterói, RJ, Brasil. *E-mail: gmacruz@vm.uff.br*

We denote by $x=(x_1,...,x_n)$ a point in $\Re^n$ while $t$ stands for the time variable. The displacement-vector is denoted by $u(x,t)=(u_1(x,t),...,u_n(x,t))$ and the thermal difference by $\theta=\theta(x,t); \lambda, \mu>0$ are Lemé's constants and $\alpha, \beta>0$ the coupling parameters.

There is an extensive literature for the study of (*) in cylindrical domains. It can be seen in [1], [3], [4], [5] and [6]. For the one-dimensional noncylindrical case, in [2], it is studied the existence, uniqueness and energy decay of weak solutions. We don't know results for (*) in n-dimensional case in noncylindrical domains.

## 2. Notations and main Results

Let us consider the real function $K(t)$ satisfying to the following conditions:

H1) $K \in C^3([0,T];\Re)$ with $K_0 = \min_{0\le t\le T} K(t)>0$.

H2) There exists a positive constant $K_1$, such that

$$\left[\mu\,\delta_i^j - (K')^2 y_i y_j\right]\epsilon_i \epsilon_j \ge K_1 |\varepsilon|^2_{\Re^n} \qquad \forall \varepsilon \in \Re^n,\ \forall y \in \Omega,\quad 0\le t\le T.$$

**Remark.** It is noted that when $(x,t)$ varies in $\hat{Q}$ the point $(y,t)$ with $y=K^{-1}(t)x$ varies in the cylinder $Q=\Omega\times\,]0,T[$. Thus, the application

$$\tau:\hat{Q}\to Q$$

given by $\tau(x,t)=(y,t)$, is a diffeomorfism. Let us consider the change of variable in (*), given by

$$\upsilon(y,t)=u(K(t)y,t)$$
$$\phi(y,t)=\theta(K(t)y,t),$$

We obtain the following mixed problem:

$$\left|\begin{array}{l}
\upsilon'' - \dfrac{\partial}{\partial y_j}\left[a_{ij}(y,t)\dfrac{\partial \upsilon}{\partial y_i}\right] - \dfrac{(\lambda+\mu)}{K^2}\nabla\, div\ \upsilon + \dfrac{\alpha}{K}\nabla\phi + b_i(y,t)\dfrac{\partial \upsilon'}{\partial y_i} \\
\quad + c_i(y,t)\dfrac{\partial \upsilon}{\partial y_i} = 0 \qquad in \quad Q \\
\phi' - \dfrac{1}{K^2}\Delta\phi + \dfrac{\beta}{K}\ div\ \upsilon' + d_i(y,t)\dfrac{\partial\phi}{\partial y_i} + \dfrac{\beta K'}{K^2}\ div\ \upsilon \\
\quad + e_i(y,t)\dfrac{\partial^2 \upsilon_j}{\partial y_j\,\partial y_i} = 0 \qquad in \quad Q \\
\upsilon = 0,\ \phi = 0 \qquad in \quad \Sigma \\
\upsilon(y,0)=\upsilon_0(y),\ \upsilon'(y,0)=\upsilon_1(y),\ \phi(y,0)=\phi_0(y),\ y\in\Omega
\end{array}\right. \qquad (**)$$

where

$$a_{ij}(y,t)=\left[\mu\delta_i^j-(K')^2\,y_i y_j\right]K^{-2},\quad b_i(y,t)=\frac{-2K'}{K}\,y_i$$

$$c_i(y,t)=\left[(1-n)(K')^2-K''K\right]K^{-2}\,y_i,\quad d_i(y,t)=\frac{-y_iK'}{K},\qquad e_i(y,t)=\frac{-\beta\,y_iK'}{K^2}.$$

Let us represent by $((.)), \|\,.\,\|$ and $(.), |\,.\,|$, the scalar product and norm in $\left[H_0^1(\Omega)\right]^n$ and $\left[L^2(\Omega)\right]^n$ respectively. We denote by $a(t,\upsilon,w)$ the bilinear form defined in $\left[H_0^1(\Omega)\right]^n$ by:

$$a(t,\upsilon,w)=\int_\Omega a_{ij}(y,t)\frac{\partial\upsilon}{\partial y_i}.\frac{\partial w}{\partial y_j}\,dy.$$

Note that $ax(t,\upsilon,w)$ is continuous, symmetric and coercive.

The main results in our work are:

**Teorema 1.** Given $u_0\in\left[H_0^1(\Omega_0)\cap H^2(\Omega_0)\right]^n, \theta_0\in H_0^1(\Omega_0)\cap H^2(\Omega_0)$, and $u_1\in\left[H_0^1(\Omega_0)\right]^n$, then there exist functions

$$u:\hat{Q}\rightarrow\Re^n \text{ and } \theta:\hat{Q}\rightarrow\Re$$

such that:

$$u\in L^\infty\left(0,T;\left[H_0^1(\Omega_t)\cap H^2(\Omega_t)\right]^n\right), u'\in L^\infty\left(0,T;\left[H_0^1(\Omega_t)\right]^n\right), u''\in L^\infty\left(0,T;\left[L^2(\Omega_t)\right]^n\right)$$
$$\theta\in L^2\left(0,T;H_0^1(\Omega_t)\cap H^2(\Omega_t)\right), \theta'\in L^2\left(0,T;H_0^1(\Omega_t)\right),$$

and they are solutions of the problem (*) in $\hat{Q}$.

**Teorema 2.** Given $\upsilon_0\in\left[H_0^1(\Omega)\cap H^2(\Omega)\right]^n, \phi_0\in H_0^1(\Omega)\cap H^2(\Omega)$, and $\upsilon_1\in\left[H_0^1(\Omega)\right]^n$, then there exist functions

$$\upsilon:Q\rightarrow\Re^n \text{ and } \phi:Q\rightarrow\Re$$

such that:

$$\upsilon\in L^\infty\left(0,T;\left[H_0^1(\Omega)\cap H^2(\Omega)\right]^n\right),\ \upsilon'\in L^\infty\left(0,T;\left[H_0^1(\Omega)\right]^n\right), \upsilon''\in L^\infty\left(0,T;\left[L^2(\Omega)\right]^n\right)$$
$$\phi\in L^2\left(0,T;H_0^1(\Omega)\cap H^2(\Omega)\right),\ \phi'\in L^2\left(0,T;H_0^1(\Omega)\right),$$

and they are solutions of the problem (**) in $Q$.

**Proof of theorem 2.** We consider $\{w_j\}$, $j = 1, 2, ...,$ a Hilbertien basis in $[H_0^1(\Omega) \cap H^2(\Omega)]^n$. We represent by $V_m = [w_1, ..., w_m]$ the subspace generate by the vectors $w_1, ..., w_m$. We find $\upsilon_m(t)$ *and* $\phi_m(t)$ *in* $V_m$, solutions of the following system of ordinary differential equations:

$$\left(\upsilon''_m, w\right) + a(t, \upsilon_m, w) - \left(\frac{(\lambda+\mu)}{K^2} \nabla\ div\ \upsilon_m, w\right) + \frac{\alpha}{K}(\nabla \phi_m, w) + \left(b_i \frac{\partial \upsilon'_m}{\partial y_i}, w\right) \tag{1}$$

$$+\left(c_i \frac{\partial \upsilon_m}{\partial y_i}, w\right) = 0 \quad \forall w \in V_m.$$

$$\left(\phi'_m, w\right) + \frac{1}{K^2}(\nabla \phi_m, \nabla w) + \left(\frac{\beta}{K}\ div\ \upsilon'_m, w\right) + \left(d_i \frac{\partial \phi_m}{\partial y_i}, w\right) \tag{2}$$

$$+\left(\frac{\beta K'}{K^2}\ div \upsilon_m, w\right) + \left(e_i \frac{\partial^2 \upsilon_{jm}}{\partial y_j\, \partial y_i}, w\right) = 0, \quad \forall w \in V_m.$$

$$\upsilon_m(0) = \upsilon_{0m} \to \upsilon_0 \ \ in\ \left[H_0^1(\Omega) \cap H^2(\Omega)\right]^n$$

$$\upsilon'_m(0) = \upsilon_{1m} \to \upsilon_1 \ \ in\ \left[H_0^1(\Omega)\right]^n$$

$$\phi_m(0) = \phi_{0m} \to \phi_0 \ \ in\ H_0^1(\Omega) \cap H^2(\Omega).$$

First Estimate

Taking $w = \upsilon'_m$ *in* $(1)$ *and* $w = \phi_m$ *in* $(2)$, we have:

$$\frac{1}{2}\frac{d}{dt}|\upsilon'_m|^2 + \frac{1}{2}\frac{d}{dt} a(t, \upsilon_m, \upsilon_m) - \frac{1}{2} a'(t, \upsilon_m, \upsilon_m) - \left(\frac{(\lambda+\mu)}{K^2} \nabla\ div\ \upsilon_m, \upsilon'_m\right) \tag{3}$$

$$+\frac{\alpha}{K}\left(\nabla \phi_m, \upsilon'_m\right) + \left(b_i \frac{\partial \upsilon'_m}{\partial y_i}, \upsilon'_m\right) + \left(c_i \frac{\partial \upsilon'_m}{\partial y_i}, \upsilon'_m\right) = 0.$$

$$\frac{1}{2}\frac{d}{dt}|\phi_m|^2 + \frac{1}{K^2}\|\phi_m\|^2 + \left(\frac{\beta}{K}\ div\ \upsilon'_m, \phi_m\right) + \left(d_i \frac{\partial \phi_m}{\partial y_i}, \phi_m\right) \tag{4}$$

$$+\left(\frac{\beta K'}{K^2}\ div\ \upsilon_m, \phi_m\right) + \left(e_i \frac{\partial^2 \upsilon_{jm}}{\partial y_j\, \partial y_i}, \phi_m\right) = 0.$$

Developing some terms of (3) and (4), we obtain:

$$-\left(\frac{(\lambda+\mu)}{K^2}\nabla\ div\ \upsilon_m, \upsilon'_m\right) = \frac{(\lambda+\mu)}{K^2}\left(div\ \upsilon_m,\ div\ \upsilon'_m\right) \tag{5}$$

$$= \frac{1}{2}\frac{d}{dt}\left(\frac{(\lambda+\mu)}{K^2}\left|div\ \upsilon_m\right|^2\right) + \frac{(\lambda+\mu)K'}{K^3}\left|div\ \upsilon_m\right|^2$$

$$\leq \frac{1}{2}\frac{d}{dt}\left(\frac{(\lambda+\mu)}{K^2}\left|div\ \upsilon_m\right|^2\right) + c_1\left\|\upsilon_m\right\|^2$$

$$\frac{\alpha}{K}\left(\nabla\phi_m, \upsilon'_m\right) = \frac{\alpha}{K}\int_\Omega \nabla\phi_m\, \upsilon'_m\, dy$$

$$-\frac{\alpha}{K}\int_\Omega \phi_m\ div\ \upsilon'_m\ dy = -\left(\frac{\alpha}{K}\ div\ \upsilon'_m, \phi_m\right) \tag{6}$$

$$\left|a'\left(t, \upsilon_m, \upsilon_m\right)\right| \leq c_2\left\|\upsilon_m\right\|^2 \tag{7}$$

$$\left|\left(b_i\frac{\partial \upsilon'_m}{\partial y_i}, \upsilon'_m\right)\right| = \left|\frac{1}{2}\int_\Omega b_i\frac{\partial\left|\upsilon'_m\right|^2}{\partial y_i}dy\right|$$

$$= \frac{1}{2}\left|-\int_\Omega \frac{\partial b_i}{\partial y_i}\left|\upsilon'_m\right|^2 dy + \int_\Gamma b_i\left|\upsilon'_m\right|^2 \eta_i\, d\Gamma\right| \tag{8}$$

$$\leq c_3\left|\upsilon'_m\right|^2$$

$$\left|\left(c_i\frac{\partial \upsilon_m}{\partial y_i}, \upsilon'_m\right)\right| \leq c_4\left(\left\|\upsilon_m\right\|^2 + \left|\upsilon'_m\right|^2\right) \tag{9}$$

$$\left|\left(d_i\frac{\partial \phi_m}{\partial y_i}, \phi_m\right)\right| = \frac{1}{2}\left|\int_\Omega d_i\frac{\partial}{\partial y_i}\left|\phi_m\right|^2 dy\right| \tag{10}$$

$$= \left|-\frac{1}{2}\int_\Omega \frac{\partial d_i}{\partial y_i}\left|\phi_m\right|^2 dy + \frac{1}{2}\int_\Gamma \left|\phi_m\right|^2 d_i\,\eta_i\, d\Gamma\right| \leq c_5\left|\phi_m\right|^2$$

$$\left|\left(\frac{\beta K'}{K^2}\ div\ \upsilon_m, \phi_m\right)\right| \le c_6\left(\|\upsilon_m\|^2 + |\phi_m|^2\right) \tag{11}$$

$$\left(e_i\ \frac{\partial^2 \upsilon_{mj}}{\partial y_j\ \partial y_i}, \phi_m\right) = \int_\Omega \phi_m\ \vec{e}.\nabla\,(div\ \ \upsilon_m)\,dy$$

$$= -\int_\Omega div\left(\phi_m\ \vec{e}\right).\ div\ \upsilon_m\ dy + \int_\Gamma \phi_m\ \vec{e}.\ div\ \upsilon_m\ d\Gamma$$

(12)

$$= -\int_\Omega div\left(\phi_m\ \vec{e}\right).\ div\ \upsilon_m\ dy\ \ where\ \ \vec{e} = (e_1, ..., e_n).$$ Thus,

$$\left|\left(e_i\ \frac{\partial^2 \upsilon_{mj}}{\partial y_j\ \partial y_i}, \phi_m\right)\right| \le \frac{1}{2K^2}\|\phi_m\|^2.$$

From (3) – (12), we have:

$$\frac{1}{2}\frac{d}{dt}\left\{\frac{\beta}{\alpha}|\upsilon'_m|^2 + \frac{\beta}{\alpha}\,a(t,\upsilon_m,\upsilon_m) + \frac{\beta}{\alpha}\left(\frac{\lambda+\mu}{K^2}\right) div\ |\upsilon_m|^2 + |\phi_m|^2\right\}$$

(13)

$$+\frac{1}{2K^2}\|\phi_m\|^2 \le c_8\left(\|\upsilon_m\|^2 + |\upsilon'_m|^2 + |\phi_m|^2\right)$$

Integrating (13) on $[0,t[$ and applying Gronwall inequality, we conclude that there is a positive constant $C$ independent of $m$ and $t$, such that

$$\frac{\beta}{\alpha}|\upsilon'_m|^2 + \frac{\beta}{\alpha}\,a(t,\upsilon_m,\upsilon_m)\ \frac{\beta}{\alpha}\left(\frac{\lambda+\mu}{K^2}\right) div\ |\upsilon_m|^2$$

(14)

$$+|\phi_m|^2 + \frac{1}{2K^2}\int_0^T \|\phi_m\|^2\,dt \le C,$$

Second Estimate

Diferentiating (1) with respect to $t$ and taking $w = \upsilon''_m$, we obtain:

$$\left(\upsilon'''_m, \upsilon''_m\right) + a\left(t, \upsilon'_m, \upsilon''_m\right) + a\left(t, \upsilon_m, \upsilon''_m\right) - \left(\frac{(\lambda+\mu)}{K^2} \nabla \; div \; \upsilon'_m, \upsilon''_m\right)$$

$$+ \frac{2(\lambda+\mu)K'}{K^3}\left(\nabla \; div \; \upsilon_m, \upsilon''_m\right) + \frac{\alpha}{K}\left(\nabla \phi_m, \upsilon''_m\right) \tag{15}$$

$$+\left(b_i \frac{\partial \upsilon''_m}{\partial y_i}, \upsilon''_m\right) + \left(b'_i \frac{\partial \upsilon_m}{\partial y_i}, \upsilon''_m\right) + \left(c_i \frac{\partial \upsilon'_m}{\partial y_i}, \upsilon''_m\right) + \left(c'_i \frac{\partial \upsilon_m}{\partial y_i}, \upsilon''_m\right) = 0$$

.

Diferentiating (2) with respect to $t$ and taking $w=\phi'_m$, we have:

$$\left(\phi''_m, \phi'_m\right) + \frac{1}{K^2}\left|\nabla \phi'_m\right|^2 - \frac{2K'}{K^3}\left(\nabla \phi_m, \nabla \phi'_m\right) + \left(\frac{\beta}{K} \; div \; \upsilon''_m, \phi'_m\right)$$

$$+\left(d_i \frac{\partial \phi'_m}{\partial y_i}, \phi'_m\right) + \left(d'_i \frac{\partial \phi_m}{\partial y_i}, \phi'_m\right) + \left(\left(\frac{\beta K'}{K^2}\right)' \; div \; \upsilon_m, \phi'_m\right) \tag{16}$$

$$+\left(e_i \frac{\partial^2 \upsilon_{jm}}{\partial y_i \partial y_i}, \phi'_m\right) + \left(e'_i \frac{\partial^2 \upsilon_{jm}}{\partial y_i \partial y_i}, \phi'_m\right) = 0.$$

Developing term terms of (15) and (16), we get:

$$a'\left(t, \upsilon_m, \upsilon''_m\right) = \int_\Omega a'_{ij} \frac{\partial \upsilon_m}{\partial y_i} \frac{\partial \upsilon''_m}{\partial y_i} \, dy \tag{17}$$

$$= \frac{d}{dt} \int_\Omega a'_{ij} \frac{\partial \upsilon_m}{\partial y_i} \frac{\partial \upsilon'_m}{\partial y_i} \, dy - \int_\Omega a''_{ij} \frac{\partial \upsilon_m}{\partial y_i} \frac{\partial \upsilon'_m}{\partial y_i} \, dy - \int_\Omega a'_{ij} \frac{\partial \upsilon'_m}{\partial y_i} \frac{\partial \upsilon'_m}{\partial y_i} \, dy \tag{18}$$

$$\frac{2(\lambda+\mu)K'}{K^3}\left(\nabla \; div \; \upsilon_m, \upsilon''_m\right) = \frac{2(\lambda+\mu)K'}{K^3} \int_\Omega div \; \upsilon_m \; div \; \upsilon''_m \; dy$$

$$- \int_\Omega \frac{2(\lambda+\mu)K'}{K^3} \left| div \; \upsilon'_m \right|^2 dy.$$

$$\frac{-2K'}{K^3}\left(\nabla\phi_m,\nabla\phi_m'\right) = \frac{d}{dt}\left(\frac{-K'}{K^3}\|\phi_m\|^2\right) + \left(\frac{K'}{K^3}\right)'\|\phi_m\|^2 \tag{19}$$

$$\frac{-\alpha K'}{K^2}\int_\Omega \nabla\phi_m\, \upsilon_m''\, dy \le C_9\left|\upsilon_m''\right|^2 + \frac{1}{4K^2}\|\phi_m\|^2 \tag{20}$$

$$\left(e_i'\frac{\partial^2\upsilon_{jm}}{\partial y_j\,\partial y_i},\phi'_m\right) = \int_\Omega e'_i\frac{\partial}{\partial y_j}\left(\frac{\partial\upsilon_{jm}}{\partial y_i}\right)\phi'_m\, dy = \int_\Omega e'_i\, div\,\frac{\partial\upsilon_m}{\partial y_i}\phi'_m\, dy$$

$$= -\int_\Omega \frac{\partial}{\partial y_i}\left(e'_i\,\phi'_m\right) div\,\upsilon_m\, dy + \int_\Gamma div\,\upsilon_m\, e'_i\,\phi'_m\,\eta_i\, d\Gamma \tag{21}$$

$$= -\int_\Omega \frac{\partial e_i'}{\partial y_i}\phi'_m\, div\,\upsilon_m\, dy - \int_\Omega e'_i\frac{\partial\phi'_m}{\partial y_i}\, div\,\upsilon_m\, dy$$

$$\le c_{10}\left(\left|\phi_m'\right|^2 + \|\upsilon_m\|^2\right) + \frac{1}{4K^2}\left\|\phi_m'\right\|^2.$$

From (15)-(21), we obtain:

$$\frac{1}{2}\frac{d}{dt}\left\{\frac{\beta}{\alpha}\left|\upsilon_m''\right|^2 + \frac{\beta}{\alpha}a\left(t,\upsilon'_m,\upsilon'_m\right) + \frac{\beta}{\alpha}\frac{(\lambda+\mu)}{K^2}\left|div\,\upsilon'_m\right|^2 + \left|\phi'_m\right|^2\right.$$

$$\left. + \frac{\beta}{\alpha}\int_\Omega a_{ij}'\frac{\partial\upsilon_m}{\partial y_i}\frac{\partial\upsilon_m'}{\partial y_j}dy + \frac{\beta}{\alpha}\int_\Omega \frac{2(\lambda+\mu)K'}{K^3}div\,\upsilon_m\, div\,\upsilon_m'\, dy - \frac{K'}{K^3}\|\phi_m\|\right\} \tag{22}$$

$$+ \frac{1}{4K^2}\left\|\phi_m'\right\|^2 \le c\left[\left\|\upsilon_m'\right\|^2 + \left|\upsilon_m''\right|^2 + \left|\phi_m'\right|^2 + \|\upsilon_m\|^2 + \|\phi_m\|^2\right]$$

Integrating (22) on $[0,t[$, and applying Gronwall inequality, we deduce:

$$\frac{\beta}{\alpha}\left|\upsilon_m''\right|^2 + \frac{\beta}{2\alpha}\rho_0\left\|\upsilon_m'\right\|^2 + \frac{\beta}{\alpha}\frac{(\lambda+\mu)}{K^2}\left|div\,\upsilon_m'\right|^2 + \left|\phi_m'\right|^2$$

$$+ \int_0^T \frac{1}{8K^2}\left\|\phi_m'\right\|^2 dt \le C \tag{23}$$

From (14) and (23), it follows that:

$$\left|\begin{array}{l} \upsilon_m'' \to \upsilon'' \quad weak\ star\ in\ L^\infty\left(0,T;\left[L^2(\Omega)\right]^n\right) \\ \upsilon_m \to \upsilon \quad weak\ star\ in\ L^\infty\left(0,T;\left[H_0^1(\Omega)\right]^n\right) \\ \dfrac{1}{K}\phi_m \to \dfrac{1}{K}\phi \quad weak\ star\ in\ L^\infty\left(0,T;H_0^1(\Omega)\right) \\ \phi_m' \to \phi' \quad weak\ star\ in\ L^2\left(0,T;H_0^1(\Omega)\right) \end{array}\right. \tag{24}$$

From (24) taking limits in (1) and (2), we have the proof of the theorem 2.

**Proof of theorem 1.**

We set

$$ {}_0(y) = u_0\left(K(0)y\right), \quad \phi_0(y) = \theta_0\left(K(0)y\right), \tag{25}$$

$$ {}_1(y) = K'(0)\nabla u_0\left(K(0)y\right).\ y + u_1\left(K(0)y\right)$$

that satisfy the hypothesis of theorem 2.

Therefore, there exist  , $\phi$ solutions of the problem (**). Using  and $\phi$ we construct $u$ and

given by:

The regularity of $u$ and  can be obtained from  , $\phi$ and the hypothesis H1). We also verify the following identities:

$$\alpha\nabla\theta = \frac{\alpha}{K}\nabla\phi \tag{27}$$

$$(\lambda+\mu)\nabla\ div\ u = \frac{\lambda+\mu}{K^2}\nabla\ div\ \upsilon \tag{28}$$

$$\mu\,\Delta u = \frac{\mu}{K^2}\Delta\upsilon \tag{29}$$

$$u'' = \upsilon'' - \frac{\partial}{\partial y_j}\left(a_{ij}\frac{\partial\upsilon}{\partial y_i}\right) + \frac{\mu}{K^2}\Delta\upsilon + b_i\frac{\partial\upsilon'}{\partial y_i} + c_i\frac{\partial\upsilon}{\partial y_i} \tag{30}$$

$$\Delta\theta = \frac{1}{K^2}\Delta\phi \tag{31}$$

$$\beta \; div \; u' = \frac{\beta}{K} \; div \; \upsilon' + \frac{\beta K'}{K^2} \; div \; \upsilon + e_i \frac{\partial^2 \upsilon_j}{\partial y_j \, \partial y_i} \tag{32}$$

$$\theta' = \phi' + d_i \frac{\partial \phi}{\partial y_i} \tag{33}$$

from (27)-(33), using the fact that $(\upsilon, \phi)$ satisfies (**), we deduce that $(u, \theta)$ satisfies (*), and hence the theorem 1 is completely proved.

## REFERENCES


1. Barreto, R.K. and Rivera, J.E.M.; Existence and exponential decay in nonlinear thermoelasticity. Nonlinear Analysis, Theory Methods and Applications, vol. 31, nº ½ (1998), pp. 149-162.
2. Caldas, C.S.; Limaco, J and Barreto, R.K.; Linear Thermoelastic system in non-cylindrical domains, Funckcialaj Ekvacioj, 42 (1999), pp. 115-127.
3. Dafermos, C.M.; On the existence and the asymptotic stability of solution to the equations of linear thermoelasticity, Arch. Rational Mech. Anal., vol. 29 (1968), pp. 241-271.
4. Henry, D.; Lopes, O. And Parisinotto, A.; Linear Thermoelasticity: asymptotic stability and essential spectrum (unpublished manuscript).
5. Lagnese, J.E.; The reachability problem for thermoelastic plates, Arch. Rat. Mech. Anal. 112, (1990). Pp. 223-267.
6. Rivera, J.E.M.; Exponential decay in thermoelasticity, Funkcialaj Ekvacioj, 35 (1992), pp. 19-30.