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A Virtual Lab for Training on Associativ Content-Addressale Memory. The Hopfield Neural Network

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Resumen Se ha creado un Laboratorio Virtual para estudiar la operación de la red de neuronas de Hopfield, de memoria asociativa, y ha sido aplicada a imágenes artificiales. La pequeña red desarrollada ha sido aplicada a cuatro grupos de imágenes de entrenamiento y remembranzas. El software permite al usuario familiarizarse con la memoria asociativa, proporciona también conocimiento sobre el funcionamiento de una red de neuronas, de modo que este laboratorio virtual puede ser utilizado como una herramienta de enseñanza y aprendizaje.

El software muestra que entre las aplicaciones de la red de Hopfield se encuentran el reconocimiento de patrones y la reconstrucción de imágenes, especialmente ésta última. Este software sirve como una introducción a redes más avanzadas y complejas.

Este reporte apunta a entender la ejecución y potencialidades de una red de neuronas, puede también estimular el interés de los estudiantes en la Cibernética. El programa incluye un conjunto de imágenes (mostrado en este reporte), sin embargo, también acepta aquellas suministradas por el usuario. Las imágenes usadas son pequeñas, pues la principal desventaja de la red de Hopfield es el hecho de que asigna una neurona a cada píxel de la imagen; sin embargo, con el poder de computo que existe hoy, esta asignación uno-a-uno, podría dejar de ser una desventaja.

Palabras clave: cibernética, inteligencia artificial, redes de neuronas artificiales, Hopfield, reconocimiento de patrones, reconstrucción de imágenes, ising, magnetismo.

Abstract A Virtual Lab to study the performance of the Hopfield's Associative Content-Addressable Neural Network has been created and applied to computer synthesized images. A small network has been developed and applied to four sets of training and remembrances presented as images. The software allows the user to become acquainted with the associative memory, it also provides knowledge on the functioning of a neural net, hence this virtual lab may be used as a training tool (teaching &).

This virtual lab makes evident that a straightforward application of the Hopfield neural networks is in the field of pattern recognition and image reconstruction, specially the latter, it also introduces the user to more advanced and complex neural networks.

This paper is aimed at understanding the performance and potentials of a neural net, it may also foster the interest of students in cybernetics.
A set of images (shown in this report) is included in the software, however it also accepts those made by the user. Small artificial images are used because the Hopfield net assigns every pixel to a neuron; this is its main drawback to industrial applications. However with the computing power existing today, this one-to-one assignment might no longer be a disadvantage

Key words: cybernetics, artificial intelligence, artificial neural networks, Hopfield, pattern recognition, Image reconstruction, ising, magnetism.

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1. Introduction

1.1 Hopfield Neural Network revival

In the past, when computers were not as powerful as today, the Hopfield network was considered only as a theoretical introduction to the field of Neural Networks (NN), because its main drawback is the fact that it requires very many neurons, plenty of interconnections, and abundant computations. Nowadays when even personal computers have become astonishing powerful, the Hopfield NN - primarily as an image reconstruction tool- recovers importance. This paper includes a brief introduction to NN, a detailed description of the Hopfield net and a report of an experiment made with small artificial images in a Virtual Lab developed as a teaching tool.

1.2 What Associative Memory is?

It is due to associative memory that when somebody sees "$E = mc^3$ ", he/she recalls that it actually is $E = mc^2$, remembers the name "Einstein" and imagines an old man with disordered long white hair, others may also remember that famous photograph where he is showing out his tongue.

1.3 What is a Neural Network?

The fruit fly is to genetics, like the Bohr's atom is to modern physics, like the Hopfield's network is to the field of neural networks.
A NN[1-6] is a set of interconnected units or neurons, see Fig.1, there are synapses (connections) with some particular strength W between neurons. The state of neuron k is S_k and W_{mn} is the strength of the synapse connecting neurons m and n. The allowed values of the states S_i may be discrete, like 1, +1 or 1, 0, +1, etc, or continuous, like S_i in the interval 0 to 1 or 1 to +1.

Generally the output of a neuron depends on the total input h_i (a weighted sum) it receives:

$$h_i = \sum_j \omega_{ij} S_j$$

the synapses W_{ij} may be excitatory, when $W_{ij} > 0$, (Neurons i and j are in Parallel states), or inhibitory, when $W_{ij} < 0$, (Neurons i and j are in Antiparallel states), and the allowed values of W_{ij} may be discrete or continuous and bounded or unbounded.
The values of the synapses W_{ij} control the performance of a NN, these have initially any random value and then they are updated as the network learns some piece of information, so that their updated values represent the learned information, this means that the information grasped by a NN is stored in the weights (synapses) connecting units (neurons).

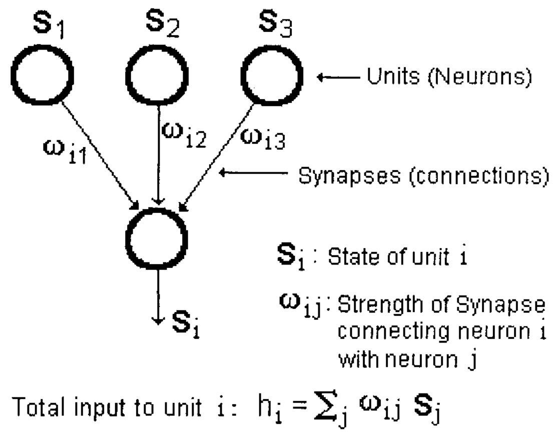

Fig. 1 A very simple Neural Network

NN have been applied mainly to pattern recognition [8-9, 14-15], both image and sound, in the industry, defence systems, speech recognition, optical character recognition, identification of fingerprints, etc.

1.4 Association from Distorted and Noisy Data

The recall of stored information by the brain relies rather on associations with previously stored data, than on the order in which the memory was acquired; with conventionally programmed computers this task requires precise knowledge of the memory address. Up to a certain degree, neural networks are able to emulate the performance of the brain. Despite the simplicity of the work of the NN reported in this paper, it can not be achieved by a conventionally programmed computer, these are not able to operate on distorted, vague and noisy data, at least not in the short time and with the straightforwardness of the NN.

1.5 Physics and Neural Networks

Physics themes [1-4] like Hamiltonians, Statistical Mechanics, finite temperature dynamics, spins in equilibrium, mean field theory, Replica-symmetry breaking, Basins of attraction, etc, have a parallel in the field of NN [10] and are very useful to understand these, for this reason physicists have a natural understanding of NN.

The Hopfield model of NN results being a variation of the Ising Spin Glass model of magnetism, where the spins updating protocol is very similar to that used in the Hopfield model, the bonds J between spins are symmetrical $J_{mn} = J_{nm}$ and the spins flip one at a time. The Hopfield model includes an Energy function, which cannot increase and whose stable states are associated with the stored patterns. The problem of NN as a memory devise is the inverse spin-glass problem[6].

1.6 Types of NN and learning protocols

There are several types [3] of NN, like Perceptrons (simple and multiplayer), Adalines, Hopfield networks, Boltzmann machine, Kohonen nets, Neocognitrons, etc.
Updating of neurons while the networks learns some piece of information may be synchronous (all S_i are changed at once) and asynchronous (only some S_i are updated every time, generally on a random basis),
Concerning the learning protocols, these may be supervised and unsupervised, and within these there are quite a few algorithms.

Researchers have developed a plethora [7-9] of NN and learning algorithms, depending on their applications, some investigators have also used genetic algorithms to develop the architecture of their networks.

1.7 Associative Content-Addressable Memory

In Associative Content-Addressable memory [11] when an input pattern is presented to the network for recognition, this evolves towards the most similar stored memory (pattern), because this most-similar pattern acts as an Attractor for the network dynamics.

The purpose of the auto-associative memory [1] is the embedding of pre-specified patterns or memories into a network for later retrieval. As can be appreciated with the software being reported in this paper, retrieval of information is possible even with partial and/or noisy information.

1.8 The three steps of a recognition process are

(1) **Training:** The NN learns some memories (patterns, images) and stores them into its synapses.

(2) **Remembrance:** The NN “sees" part of a learned pattern (memory), this partial "view" may even be distorted and noisy.

(3) **Recognition:** The NN associates "seen" information either Asynchronously (Randomly updating a neuron at a time) or synchronously (Updating all neurons simultaneously at once), to reconstruct the corresponding memory (pattern).

2.0 The Hopfield Neural Network

2.1 General Features [11-13]

- There is only a single layer of neurons (other models have more layers).
- There are feedback connections from each neuron to every other neuron.
- There are no connections from a neuron to itself.
- The weights (strengths) on the connections (synapses) are symmetrical.
- There is an Energy function E.
- The gain of the neurons is controlled by a Sign function.
- The Energy landscape has many local minima, being this the main drawback in terms of engineering applications.

2.2 Learning in the Hopfield Network

There are N interconnected binary neurons, then there are N x N synapses W_{ij}, of the which a number N (N-1)/2 are pairs of symmetric connections $W_{ij} = W_{ji}$.
Every Neuron Si may be either On or Off, this is: Si = +1 or Si = -1
Assume the number of patterns (memories, images) to be stored is p and are given by $\xi_i^\mu = \pm 1 \quad \mu = 1,2,3,\ldots p$

Each pattern has N bits (±1), one for each neuron i.

The usual way to change the links W_{ij} so as to learn the input patterns is the Hebb learning rule, which in the case of pattern u is given by:

$$w_{ij} = \frac{1}{N}\sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu \qquad i,j = 1,2,\ldots,N$$

where u = 1, 2, ... , p and ξ_i^u is the bit i of pattern u to be stored, w_{ij} is the synapse between neuron i and j. The Hebb prescription -as it is also known- is a one-shot learning rule, which means that learning takes place in a single operation, with more sophisticated learning algorithms a number of iterations-updates are necessary for the information to be grabbed by the network.

The new state S_i of neuron i, this is the network dynamics, is given by

$$S_i^\mu(t+1) = \Theta\left[\sum_{j=1}^{N} w_{ij}\, S_j^\mu(t) - T\right]$$

where T is a threshold (usually T = 0), and $\Theta(x)$ is the Sign function

$$\Theta(x) = \begin{cases} +1 & when \quad x \geq 0 \\ -1 & when \quad x < 0 \end{cases}$$

The Energy function is

$$E = -\frac{1}{2}\sum_{ij} \omega_{ij}\, S_i\, S_j$$

in Spin Glasses this is the Magnetic Interaction Energy and involves all the spins in the system, in neural networks this is an abstract quantity associated to the information content of the net. The Energy surface has local minima at the points $S_i = \xi_i^{\mu}$. The Energy is used during learning when there are iterations involved, in every iteration E goes down as the NN learns.

2.3 Storage Capacity

In the current case patterns are 15x15 pixels, which means that the number of neurons (one for each pixel) is N = 15x15 = 225.

According to Hopfield, if there are small errors in the stimuli, the maximum number of patterns p_{max} that can be stored in a network with N neurons, is $S_i = \xi_i^{\mu}$ $p_{max} = 0.15$ N, ($p_{max} = 33.75$, for N = 225), he reached to this conclusion by means of computer simulations. Then Amit et al [16] through the theoretical Replica Method showed that p_{max} = 0.138 N, (p_{max} = 31.05, when N = 225). Mc Eliced et al [17] concluded that for p < N / (4 ln N), the Hopfield NN is able to reconstruct all learned patterns with no error. In the current case:

$$p = \frac{N}{4\, Ln\, N} = 10.38$$

which means that the number of patterns must be at most 10, but this is as well as the patterns are fully orthogonal, in real life patterns are far from being sharing this feature and this number is greatly reduced, for this reason the network of this work operates on at most 4 patterns.
For orthogonal patterns ξ_i^{μ} and ξ_i^{ν}, their scalar (dot) product satisfies:

$$\sum_{i=1}^{N} \xi_i^{\mu}\, \xi_i^{\nu} = 0 \qquad for \qquad \mu \neq \nu$$

3.0 The Experiments

A Virtual Lab (software) to simulate the Hopfield NN has been created and four independent experiments were carried out. In every experiment the network is first trained with a set of image inputs, these are learned as memories, then a number of remembrances (stimuli) is presented one at a time to the net and this, by means of association, recalls the more similar image memorized during training.

3.1 Experiment 1

Fig. 2 shows the set of (four) memories (input set) the network is trained with (square, rhombus, triangle, arrow). As it can be seen in figs A1 through A10, the 10 presented stimuli (remembrances) are not so similar to the images in the training set, they are distorted, sometimes vague and include noise, however, the network after being stimulated reconstructs the most similar memory (Recognition).

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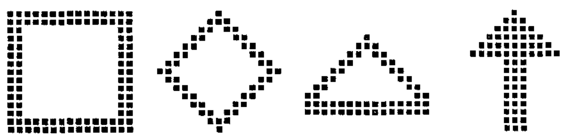
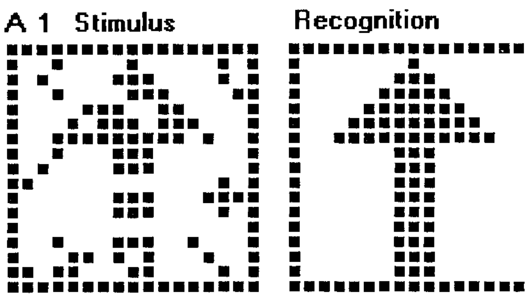
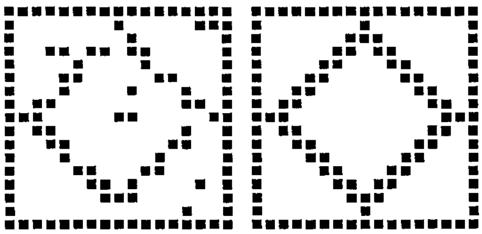


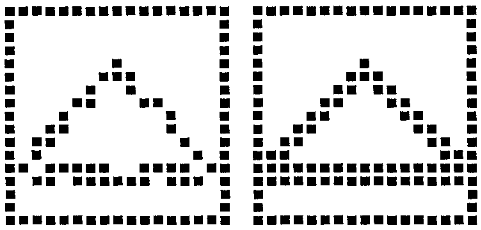
Fig 2. Training set A: 4 objects



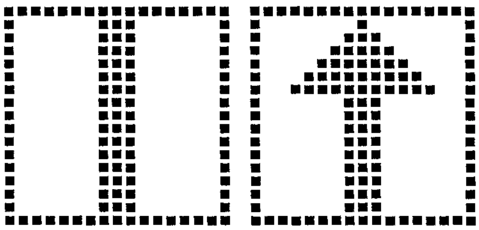
A 2 Stimulus Recognition



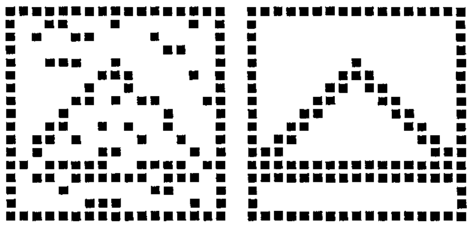
A 3 Stimulus Recognition



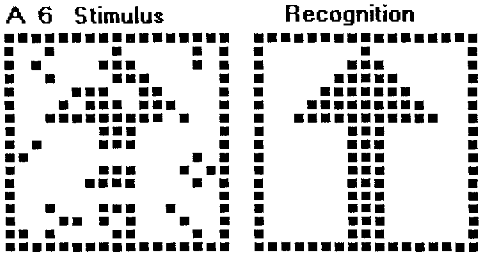
A 4 Stimulus Recognition



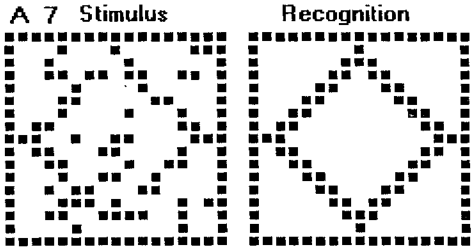
A 5 Stimulus Recognition



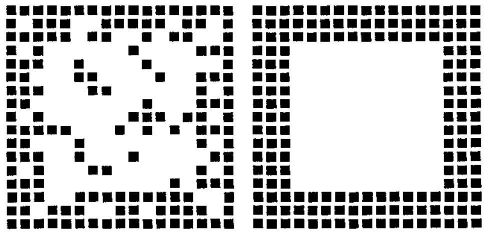
A 6 Stimulus Recognition



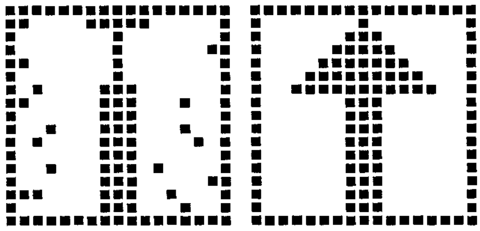
A 7 Stimulus Recognition



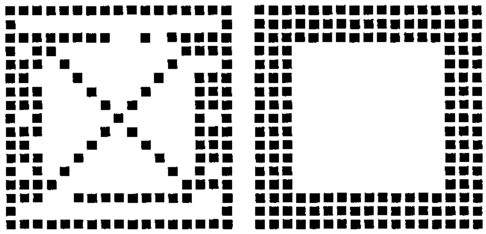
A 8 Stimulus Recognition



A 9 Stimulus Recognition



A 10 Stimulus Recognition



Note that even though the stimuli are confusing, noisy and distorted images, the neural network reconstructs the original objects.

3.2 Experiment 2

The Network was trained with two images (Fig. 3), a happy face and a plus sign, then it was motivated with the stimuli in images B1 through B10, the reconstructed images appear as Recognition.

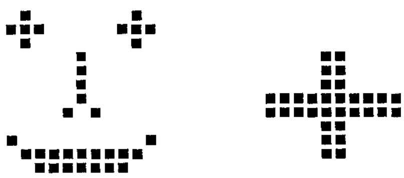
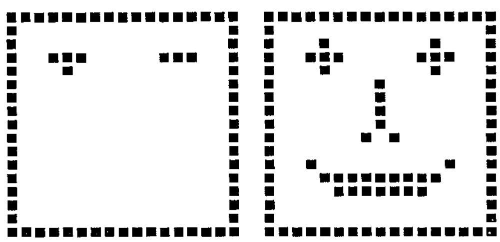
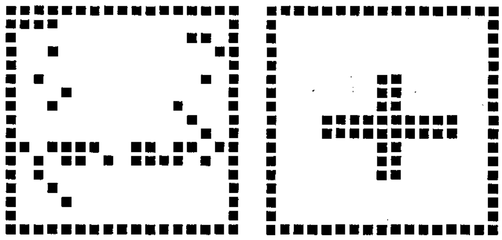


Fig. 3 Training set B 2 objects

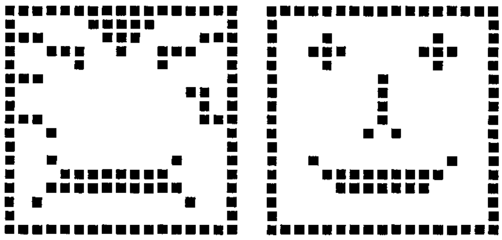
B 1 Stimulus Recognition



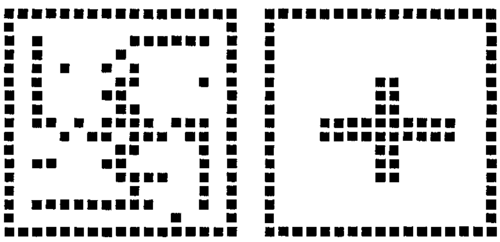
B 2 Stimulus Recognition



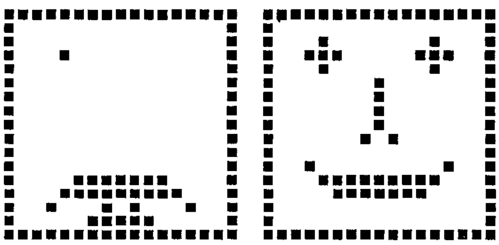
B 3 Stimulus Recognition



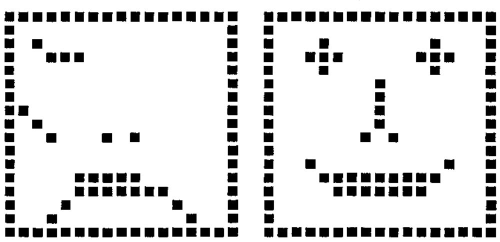
B 4 Stimulus Recognition



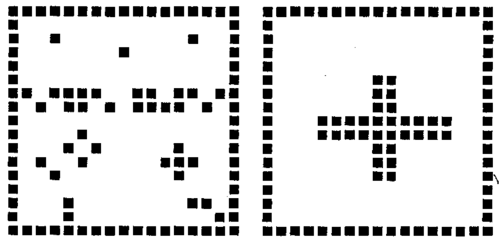
B 5 Stimulus Recognition



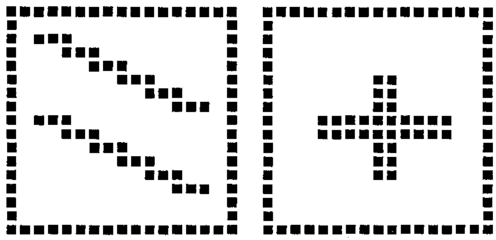
B 6 Stimulus Recognition



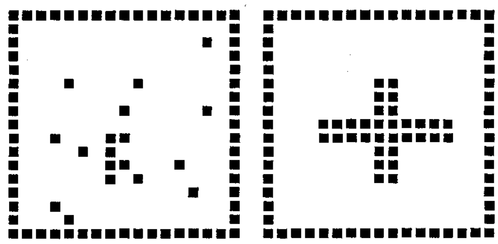
B 7 Stimulus Recognition



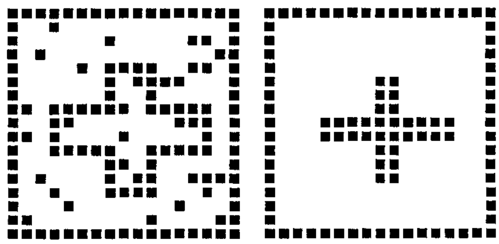
B 8 Stimulus Recognition



B 9 Stimulus Recognition



B 10 Stimulus Recognition



It can be seen that notwithstanding the remembrances (stimuli) are in many case vague, distorted and noisy, the network is able to make the right association. In all cases the stimuli are sad faces or part of them, however the network has reconstructed the original happy faces of the input.

Stimulus B2 as well as B7 show part of a horizontal line, but not in the position of that of the cross, however the network recognises the cross in its correct (original) position.

In fig. B4, the system discards the swastika (which is a cross) and reconstructs the original cross. In Fig. B8 the stimulus are two diagonal lines and the net seems to "think" that any two lines must appear crossing themselves. In computer graphics a straight line is not only a succession of points but also a succession of line-segments.

Notice in fig. B10, that the stimulus is not the plus sign of the corresponding training set, but rather the closed line around it, apart of noise, there are only 8 scattered dots belonging to the original plus sign, however the network associates it to the correct memory.

3.3 Experiment 3

Fig. 4 shows the input set for experiment 3, the input set (3 images) now is that used in experiment 2 (Fig 3) but includes an additional memory, the square. In this case, the 10 stimuli (remembrances) were those from B1 to B10, all were successfully reconstructed (recognized), except those shown in C7, C8 and C10. Notice that reconstructions C7 and C8 are both a combination of the happy face and plus sign, and that reconstruction C10 includes parts of the three training memories.

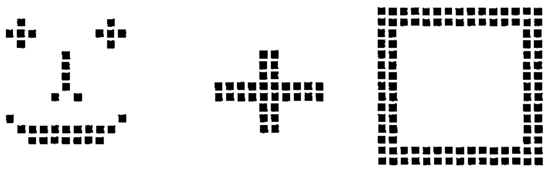
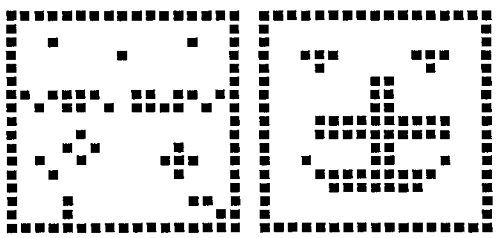
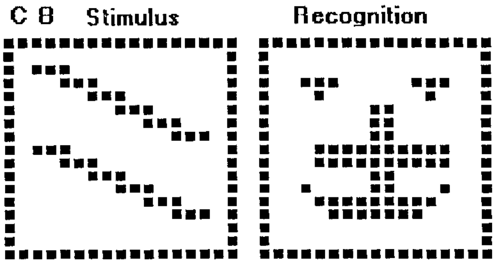


Fig 4 Training set C: 3 objects

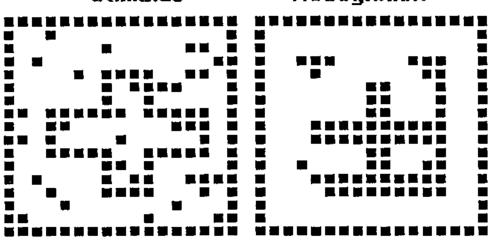
C 7 Stimulus Recognition



C 8 Stimulus Recognition



C 10 Stimulus Recognition



3.4 Experiment 4

The input set of four images is shown in Fig. 5, the 10 remembrances used as stimuli are those images shown in B1 through B10. In this case only two inputs (B3 and B4) were successfully recognized, the other eight were not. As examples of the ill-recognized memories, recognitions D8 and D10 are shown.

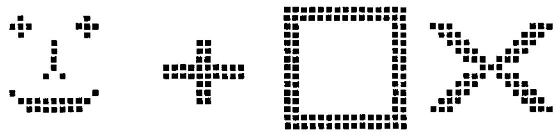
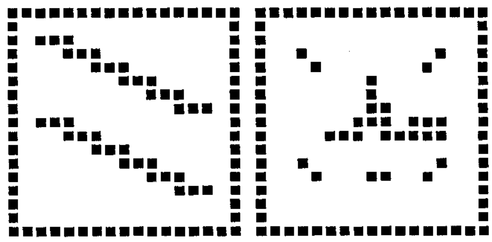
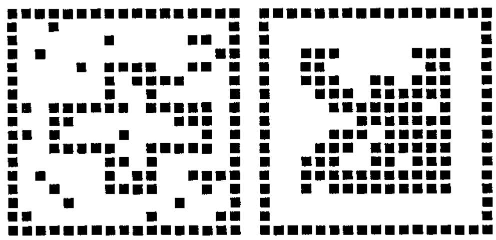


Fig 5 Training set D: 4 objects

D 8 Stimulus Recognition



D 10 Stimulus Recognition



4. Conclusions

A Virtual Lab to be used as a teaching tool on the Hopfield model for associative memory has been developed. This software may be used to learn about associative memory in general, to understand what a NN is and to see that a network like that of Hopfield can be used in pattern recognition and image reconstruction, even with distorted, noisy and vague images.

Based on this NN software development experience, larger softwares to operate on more powerful computers may be developed.

In the figures shown the recognition has been Synchronous (ordered neuron updates), the Asynchronous case (random updates) is not shown, however the results are the same but take longer because being it random, many neurons may be activated more than once and this takes computer time.

The experiments have shown that when the network tries to learn input patterns that are somehow related among them (mathematically speaking, not orthogonal), the recognition presents problems, the system becomes confused and memories too close to each other tend to merge [11], a common phenomenon among humans and maybe also among animals.

Acknowledgement

The development of this Neural Network software would not have been possible without the help of Miss Marlene Gonzalez Reyes, who created the computer synthetized objects and helped with the computer programming.

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